

Assignment #3

Due: 11:59pm on Tuesday, Nov. 7, 2023

Submit via Gradescope (each answer on a separate page) code: 7DVJKY

Problem 1. Liquidations. In Lecture 9 we discussed lending protocols. Our friend Bob uses a lending protocol to borrow ETH and AXS against his USDC collateral. He has the following debt position:

$$+1000 \text{ USDC}, \quad -0.2 \text{ ETH}, \quad -20 \text{ AXS}.$$

The collateral factor for these assets are: 0.9, 0.8., 0.7 for USDC, ETH, and AXS, respectively.

- Suppose the current exchange rates for USDC and AXS are 1500 USDC/ETH and 100 AXS/ETH. What is the **health** of Bob's debt position as defined in the lecture (slide 29)? Does Bob's collateral need to be liquidated?
- Suppose the exchange rates for AXS changes to 48 AXS/ETH (making AXS more valuable than before) and the other exchange rates remain unchanged. What is the **health** of Bob's debt position now? Does Bob's collateral need to be liquidated?
- For parts (a) and (b), if you concluded that Bob's collateral needs to be liquidated, then consider a liquidator who is willing to clear Bob's ETH debt in exchange for Bob's USDC collateral at a rate of 1520 USDC/ETH. How much USDC will Bob lose from his collateral so that his debt position becomes healthy again. Please round your answer to the smallest integer that makes Bob's debt position healthy.

Problem 2. Slippage. In Lecture 10 we discussed the constant product market maker $xy = k$ used by Uniswap. Suppose Alice wants to buy Δx type X tokens from Uniswap. We showed in the lecture that, assuming no fees ($\phi = 1$), she would have to send $\Delta y = y \cdot \Delta x / (x - \Delta x)$ type Y tokens to Uniswap to maintain the $xy = k$ invariant (see also this [short writeup](#)). Therefore, the exchange rate Alice is getting from Uniswap is

$$\frac{\Delta y}{\Delta x} = \frac{y}{x - \Delta x}.$$

In the open market, the exchange rate is some value p . Let us define the *slippage* s as

$$s = \frac{(\Delta y / \Delta x) - p}{p}.$$

This measures the difference in exchange rate between Uniswap and the open market (hence the name *slippage*). If $s = 0$ then the Uniswap exchange rate is the same as on the open market. If $s > 0$ then the Uniswap exchange rate is worse.

Show that the slippage s is always positive, and is approximately $s \approx \Delta x / x$, assuming x is much larger than Δx . Use the fact that we know that $p = y / x$ (by slide 29), and that for a small $\epsilon > 0$ we have $1 / (1 - \epsilon) \approx 1 + \epsilon$. Your derivation shows that the exchange rate in Uniswap is always worse than on the open market, however, the larger the liquidity pool, the larger x is, and therefore the smaller the slippage $\Delta x / x$, for a fixed Δx .

Problem 3. Sandwich attacks. Consider two assets X and Y on the Uniswap exchange. The X pool contains x tokens of type X and the Y pool contains y tokens of type Y . Recall that Uniswap v2 ensures that $x \cdot y = k$ for some constant k . Suppose that Uniswap charges no fees (i.e., $\phi = 1$).

Alice submits a transaction Tx that sends βx tokens of type X to Uniswap, for some $\beta \geq 0$ (here βx means β times x). When Tx executes, Uniswap will send back γy tokens of type Y to Alice, where $\gamma = \frac{\beta}{1+\beta} \in [0, 1)$. This ensures that the constant product is maintained.

Searcher Sam sees Alice's transaction in the mempool and decides to execute a sandwich attack. Sam issues two transactions Tx_1 and Tx_2 and arranges with the current block proposer that Tx_1 will appear before Alice's transaction in the proposed block and Tx_2 will appear after.

- a. Suppose Sam's Tx_1 sends ϵx tokens of type X to Uniswap, for some $\epsilon \geq 0$. Now, when Alice's Tx executes, she will receive back $\gamma' y$ tokens of type Y . What is γ' as a function of ϵ and β ? Is she getting more or less tokens of type Y than before?
- b. Does Sam's profit increase or decrease with the amount he spends in Tx_1 ? In other words, is a larger ϵ better or worse for Sam?
- c. The validator Victor who is proposing the block sees both Sam's transactions. Victor realizes that he can issue both transactions itself and keep all the profit to itself. However, Victor has no X tokens, as needed for Tx_1 . Can Victor mount the sandwich attack without Sam?
- d. In Lecture 11 we discussed MEV-boost. Can Alice use MEV-boost to protect herself from these shenanigans by Sam and Victor?