# Decentralized Exchanges (DEXs)

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## Why try to "decentralize" an exchange?

- Composability (brief rant)
- Credibly Neutral
- Security
- Global Reach

## What is a DEX?

A decentralized exchange (or DEX) is an online marketplace where transactions occur directly between participants, without the aid of any trusted intermediaries.

## Key Properties

- Composable / Programmable
- Credibly Neutral
- Non-Custodial
- Permissionless

# First Approach: Order Book Based DEXs

# Order Book Based DEXs

## The Relayer Model

- Matching is done **off-chain** by a centralized "Relayer"
    - The relayer crafts a transaction off-chain that resembles an atomic-swap, then submits it to the blockchain
- Trade settlement is done on-chain

Many examples of DEXs that initially worked this way:

- 0x protocol
- EtherDelta
- Kyber
- Airswap

# Order Book Based DEXs

## Limitations of the Relayer Model

- Less programmable/composable
- Depends on the presence of a centralized party
- Peer-to-peer —hard to bootstrap liquidity
- It's expensive with today's blockchains because of gas

**Great resource:**
Front-Running, Griefing, and the Perils of Virtual Settlement,
by Will Warren

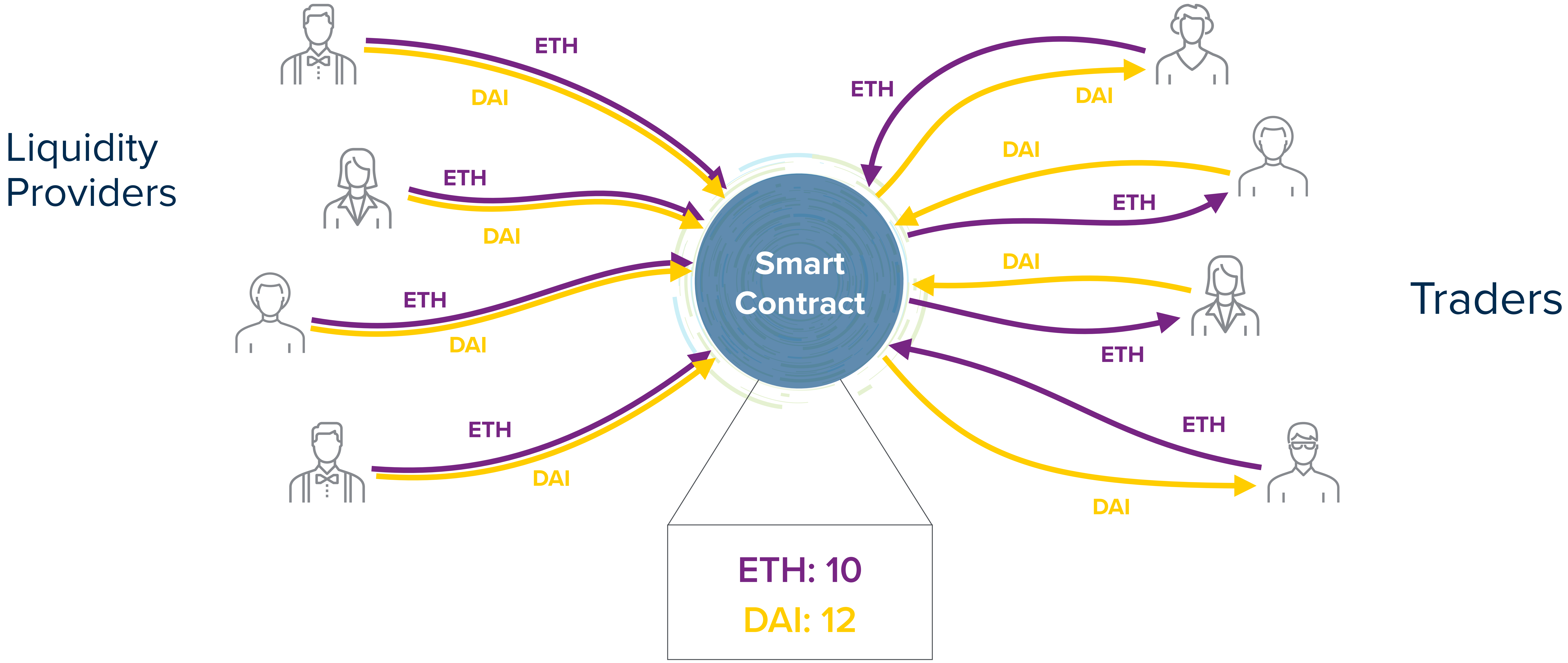
# Is there a simpler way to build a DEX?

## A Bit of History: Automated Market Makers (AMMs)

- Pricing shares in prediction markets — Hanson's Market Scoring Rules
  - Also used to price online ads
- Idea first explored in crypto in 2016 by:
  - Vitalik Buterin — reddit post
- Then generalized by Alan Lu and Martin Koppelman:
  - Blogpost: Building a Decentralized Exchange in Ethereum

# High Level Aspiration

## Two-Sided Marketplace

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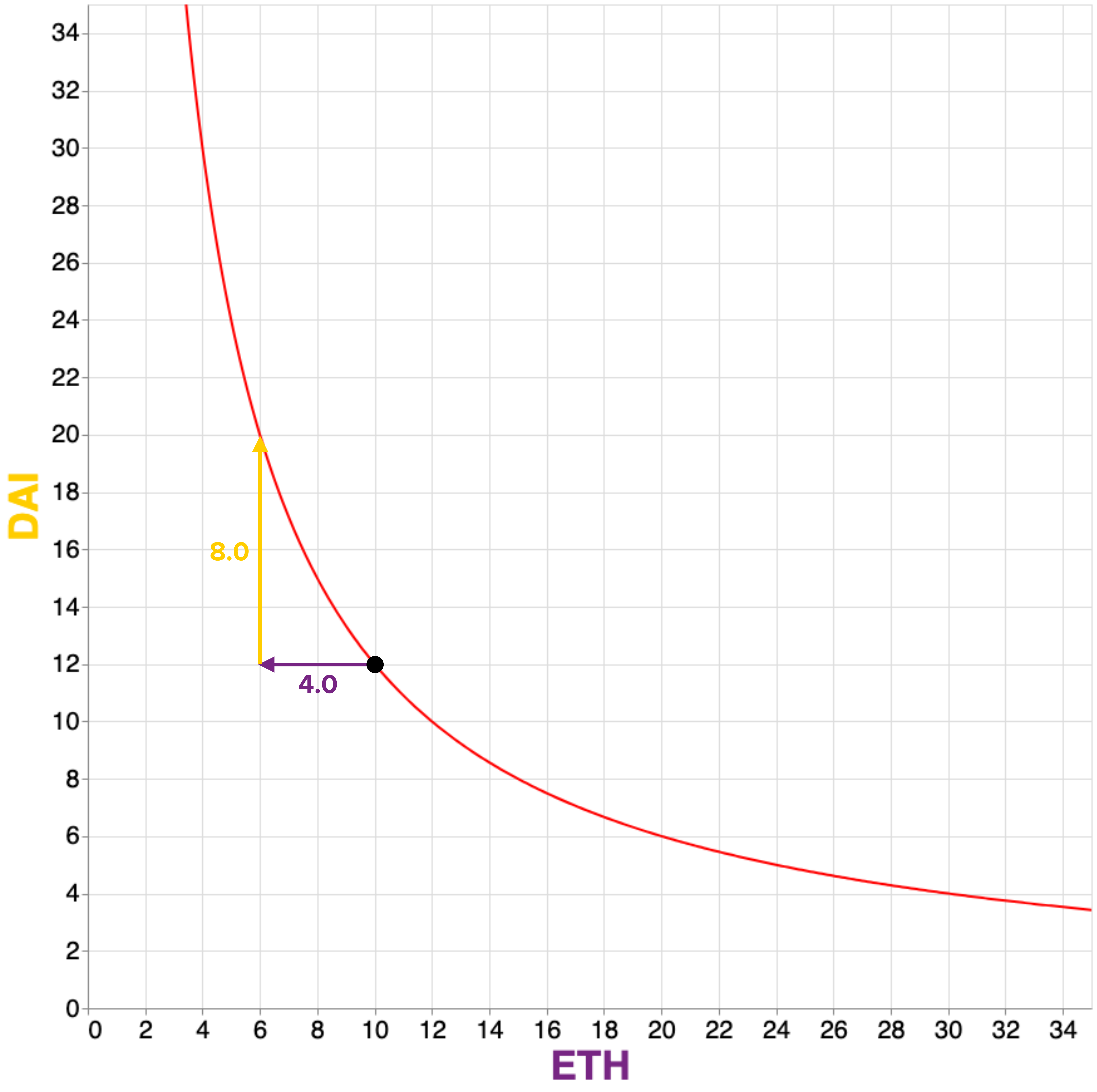
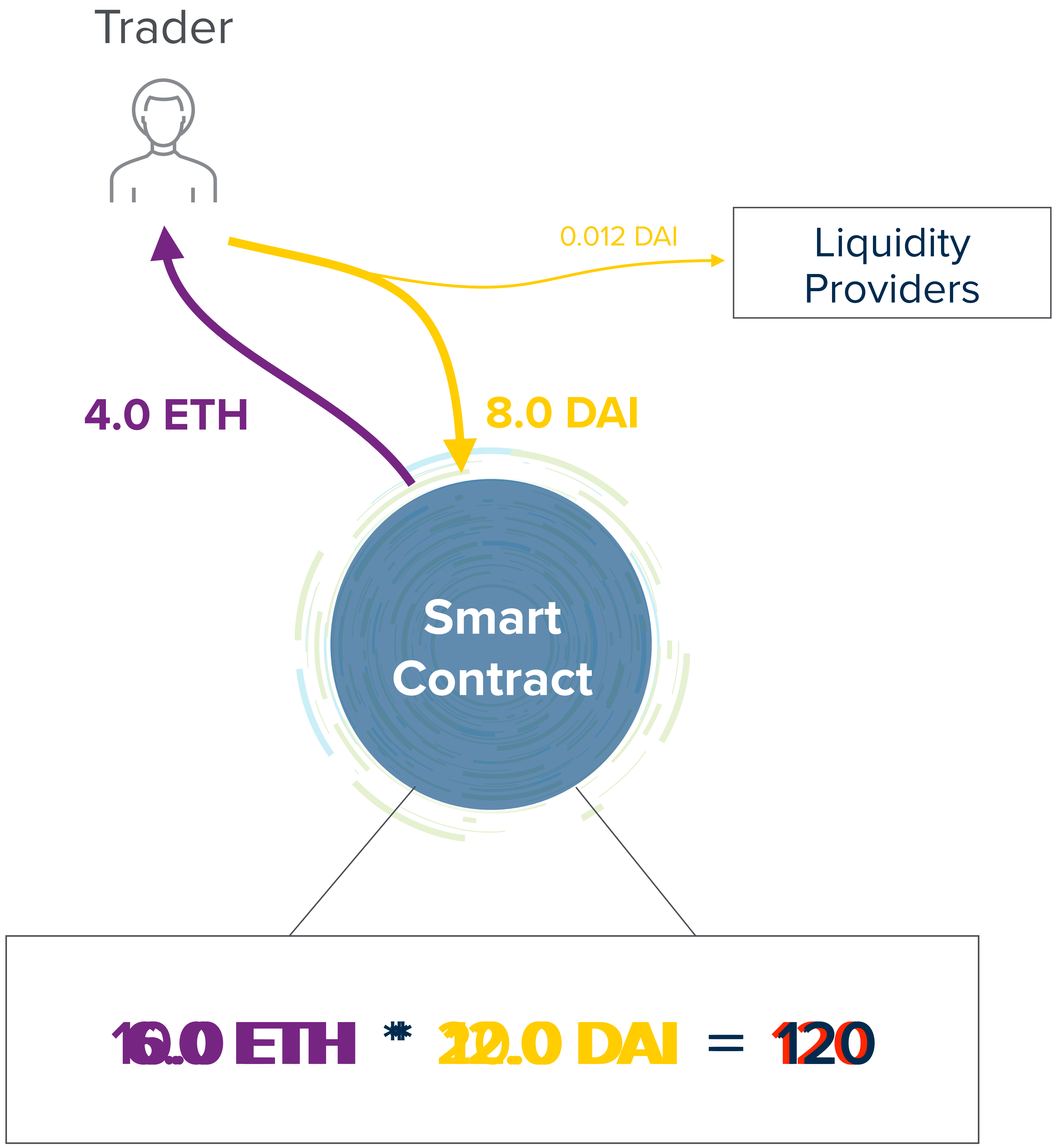
$$xy = k$$


DAI
ETH
8.0
4.0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34


Trader
0.012 DAI
Liquidity Providers
4.0 ETH
8.0 DAI
Smart Contract
10.0 ETH * 12.0 DAI = 120

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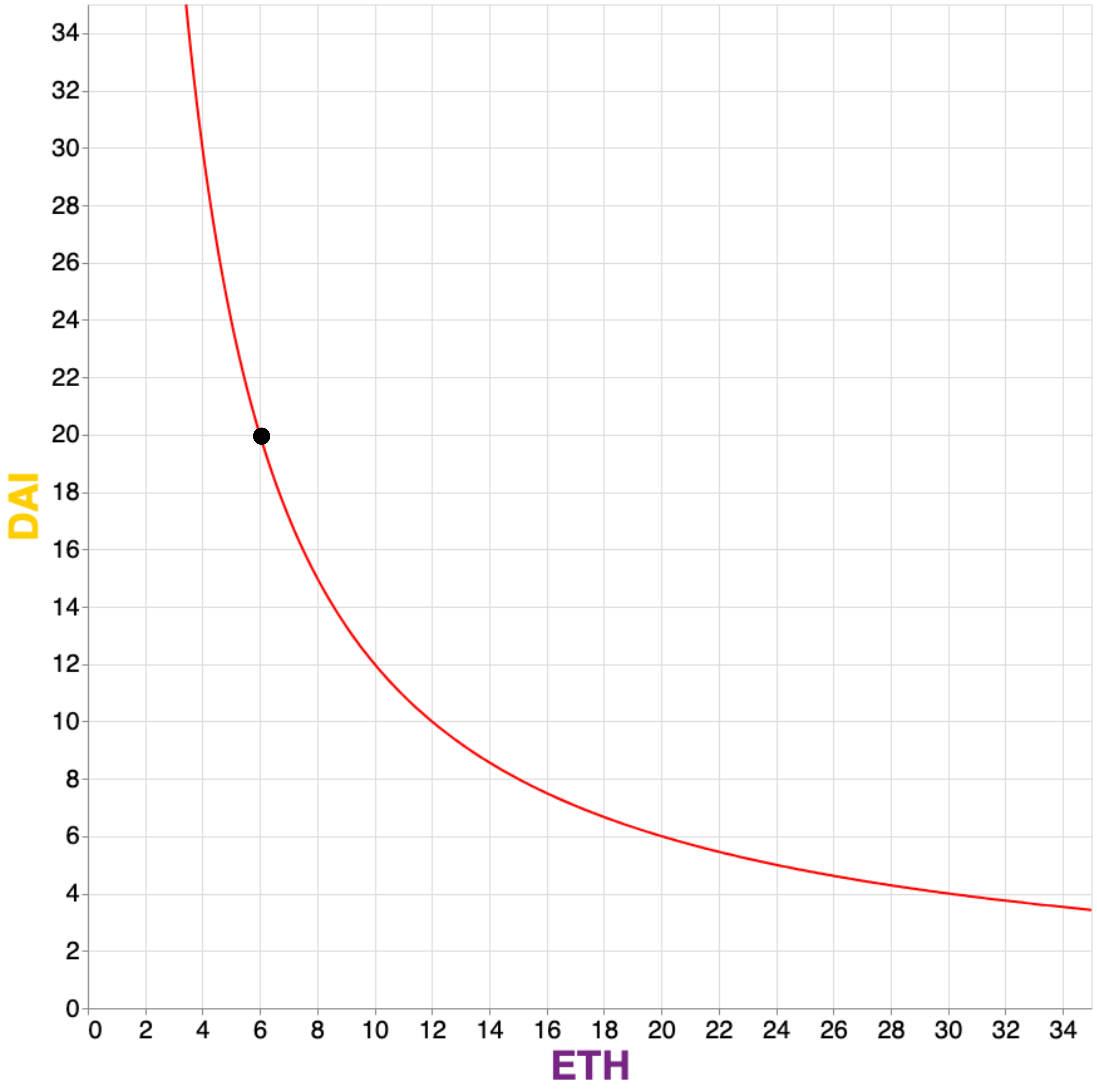
# Uniswap V2

Demo: app.uniswap.org

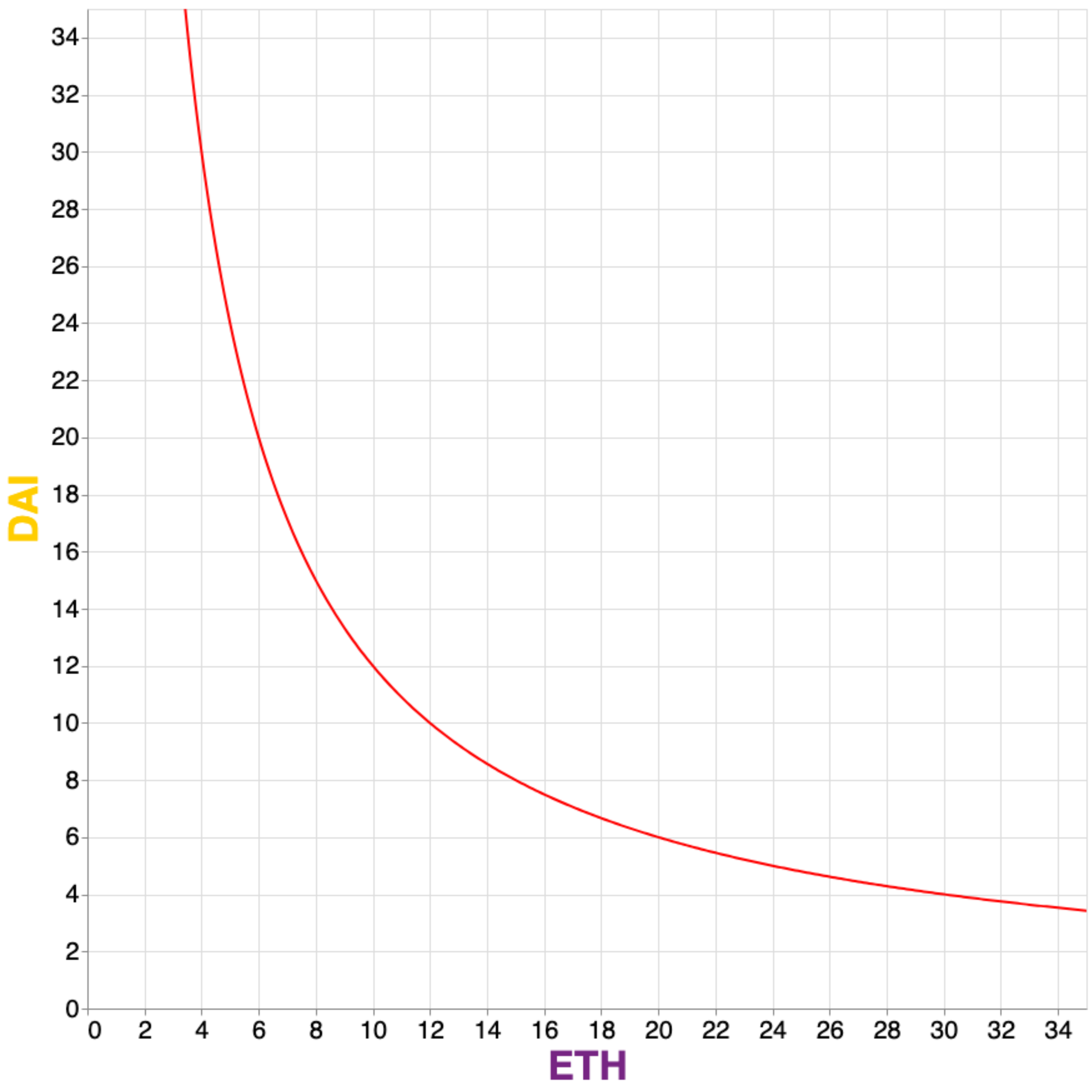
## *Invariant:* $xy = k$




## Simple Pricing Rule

$$(x - \Delta x)(y + \Delta y) = k$$

$$xy = k$$

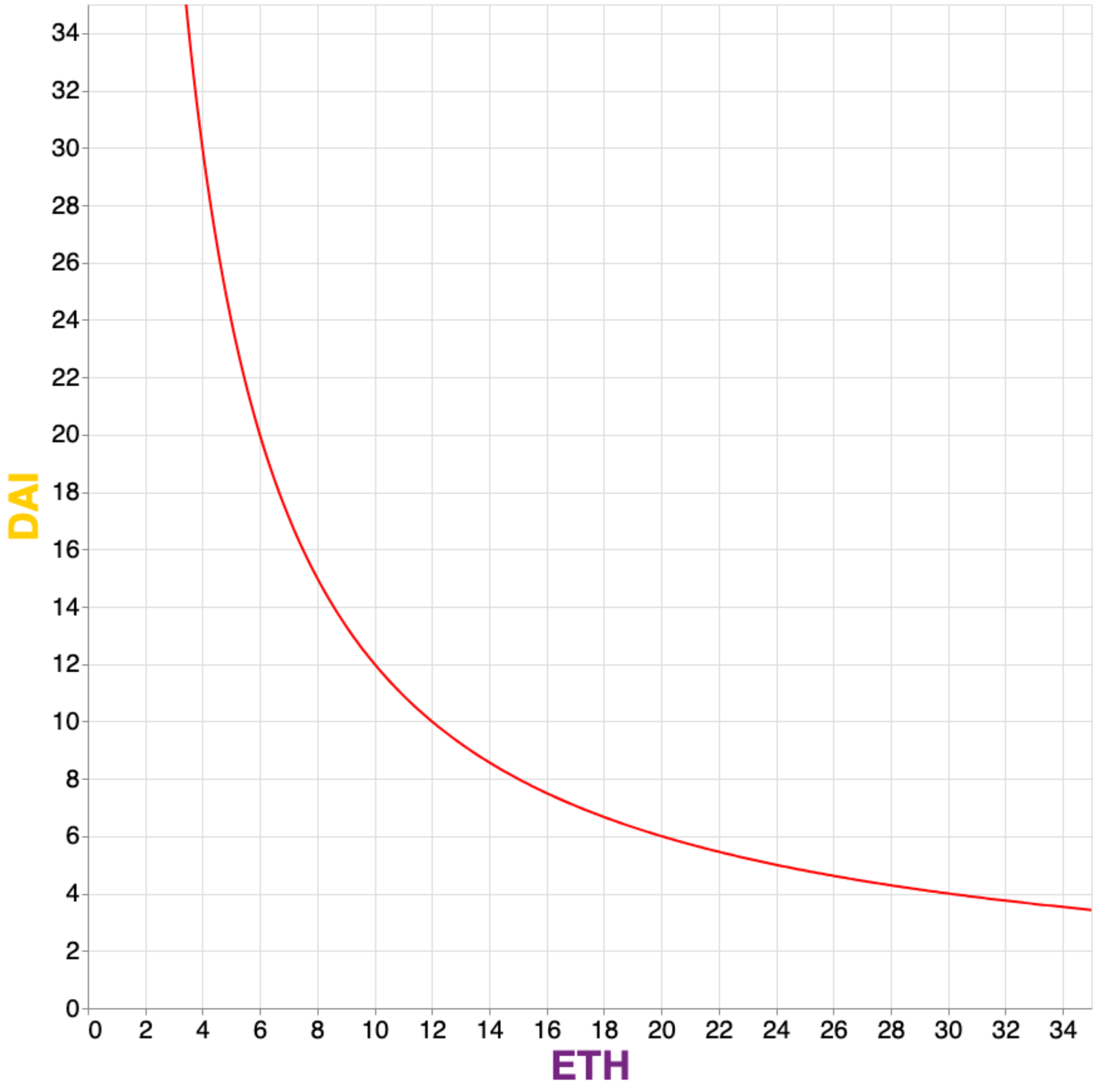



## Simple Pricing Rule

$$(x - \Delta x)(y + \phi \Delta y) = k$$

where $(1 - \phi)$ is the percentage fee that is paid to liquidity providers, and where $\Delta x > 0$ and $\Delta y > 0$.

$$xy = k$$




## Simple Pricing Rule

$$(x - \Delta x)(y + \phi \Delta y) = k$$

$$\phi \Delta y = \frac{xy}{x - \Delta x} - y$$

$$= \frac{xy - y(x - \Delta x)}{x - \Delta x}$$

$$= \frac{\cancel{xy} - \cancel{xy} - y\Delta x}{x - \Delta x}$$

$$\Delta y = \frac{1}{\phi} \cdot \frac{y \Delta x}{x - \Delta x}$$

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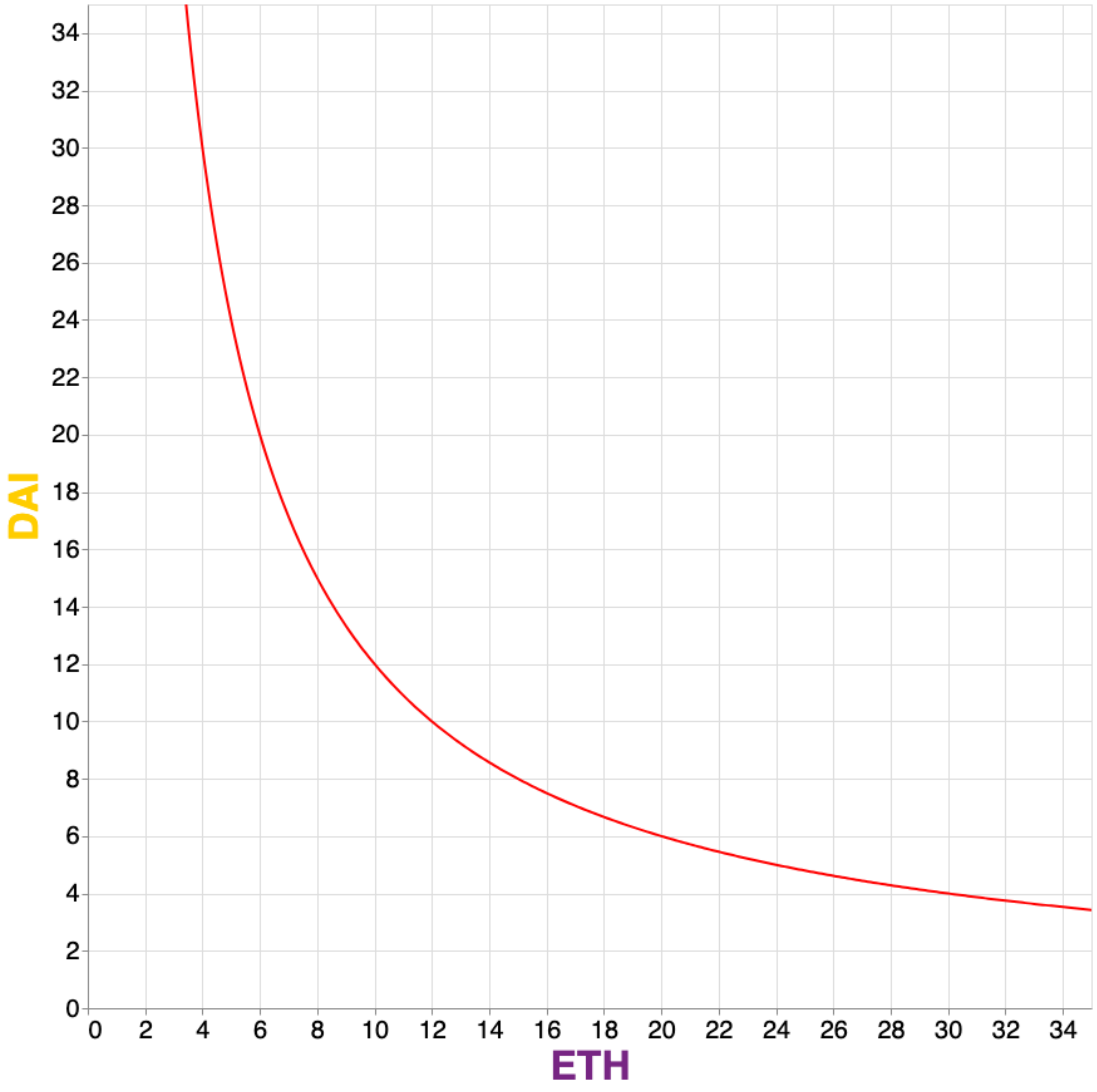
$$xy = k$$




## Simple Pricing Rule

$$\boxed{\Delta y = \frac{1}{\phi} \cdot \frac{y \Delta x}{x - \Delta x}}$$

This rule specifies the price of *buying* $\Delta x$ in terms of $y$.

A similar exercise (swapping $x$s and $y$s) produces a rule that specifies the price of *selling* $\Delta x$ in terms of $y$:

$$\boxed{\Delta y = \frac{y \phi \Delta x}{x + \phi \Delta x}}$$

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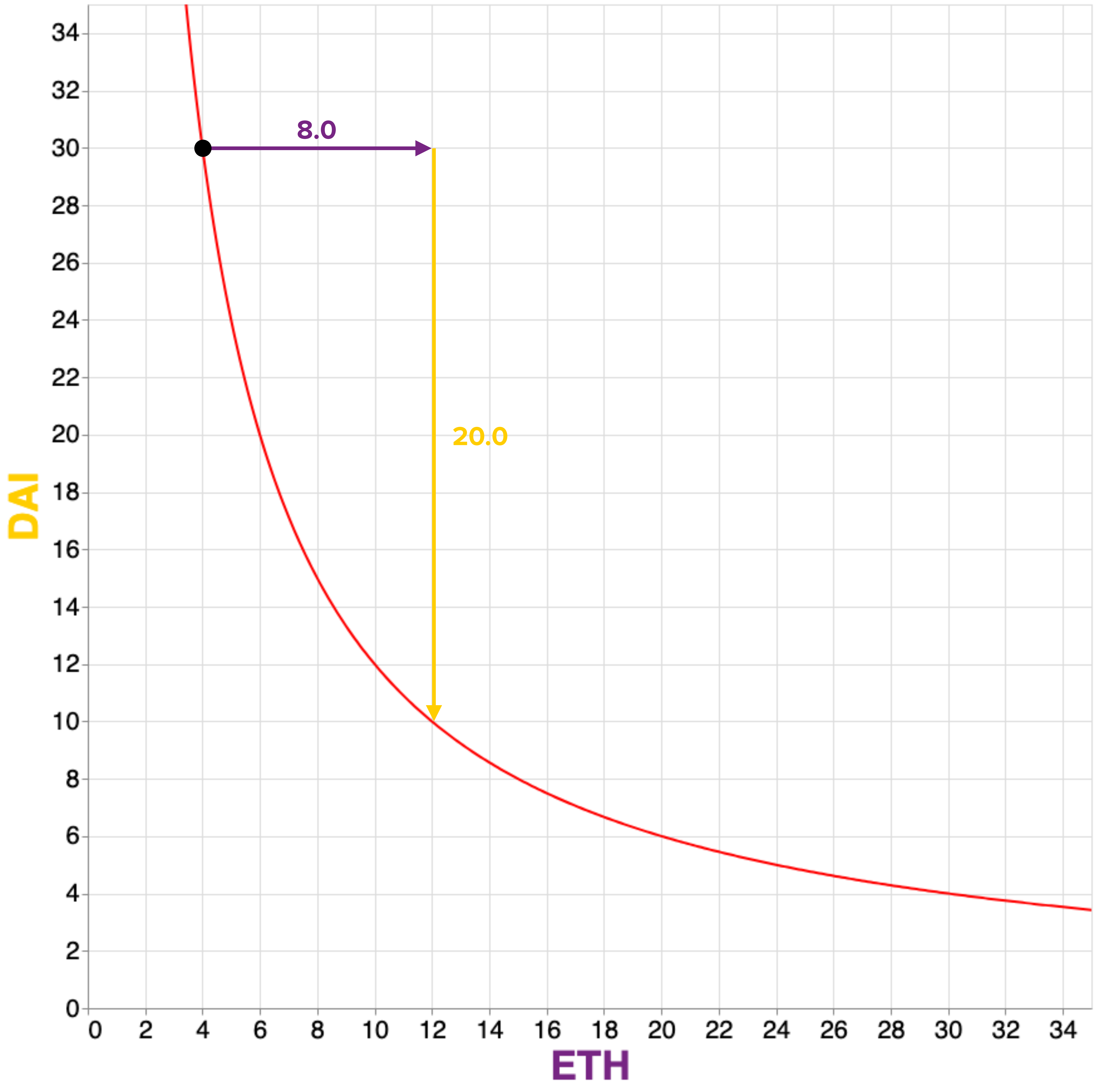
$$xy = k$$




## Simple Pricing Rule

Example where the contract contains **4.0 ETH** and **30.0 DAI** and charges a fee for liquidity providers of 30 bps.

$$\Delta y = \frac{y\phi\Delta x}{x + \phi\Delta x}$$

$$\Delta y = \frac{30 * 0.997 * \Delta x}{4 + 0.997 * \Delta x}$$

Say a trader wants to *sell* **8.0 ETH** to the contract. How much **DAI** should she get in return?

$$\Delta y = \frac{30 * 0.997 * 8}{4 + 0.997 * 8} = 19.98$$

(The fee to liquidity providers is 0.02.)

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# In the Wild: Uniswap

## Selling $x$ for $y$

$$\Delta y = \frac{y\phi\Delta x}{x + \phi\Delta x}$$

```
    // given an input amount of an asset and pair reserves, returns the maximum output amount of the other asset
    function getAmountOut(uint amountIn, uint reserveIn, uint reserveOut) internal pure returns (uint amountOut) {
        require(amountIn > 0, 'UniswapV2Library: INSUFFICIENT_INPUT_AMOUNT');
        require(reserveIn > 0 && reserveOut > 0, 'UniswapV2Library: INSUFFICIENT_LIQUIDITY');
        uint amountInWithFee = amountIn.mul(997);
        uint numerator = amountInWithFee.mul(reserveOut);
        uint denominator = reserveIn.mul(1000).add(amountInWithFee);
        amountOut = numerator / denominator;
    }
```

## Buying $x$ for $y$

$$\Delta y = \frac{1}{\phi} \cdot \frac{y\Delta x}{x - \Delta x}$$

```
    // given an output amount of an asset and pair reserves, returns a required input amount of the other asset
    function getAmountIn(uint amountOut, uint reserveIn, uint reserveOut) internal pure returns (uint amountIn) {
        require(amountOut > 0, 'UniswapV2Library: INSUFFICIENT_OUTPUT_AMOUNT');
        require(reserveIn > 0 && reserveOut > 0, 'UniswapV2Library: INSUFFICIENT_LIQUIDITY');
        uint numerator = reserveIn.mul(amountOut).mul(1000);
        uint denominator = reserveOut.sub(amountOut).mul(997);
        amountIn = (numerator / denominator).add(1);
    }
```

UniswapV2Library.sol


Quick Demo: https://app.uniswap.org/

# How to Think about an AMM's Price

Price is the ratio between assets (e.g. **DAI**) paid and assets (e.g. **ETH**) received.

If I pay **100 DAI** for **4 ETH**, then my price per ETH is 25 DAI.

In our notation, this is given by $|\Delta y/\Delta x|$.

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Selling $x$ for $y$

$$\Delta y = \frac{y\phi\Delta x}{x + \phi\Delta x}$$

Buying $x$ for $y$

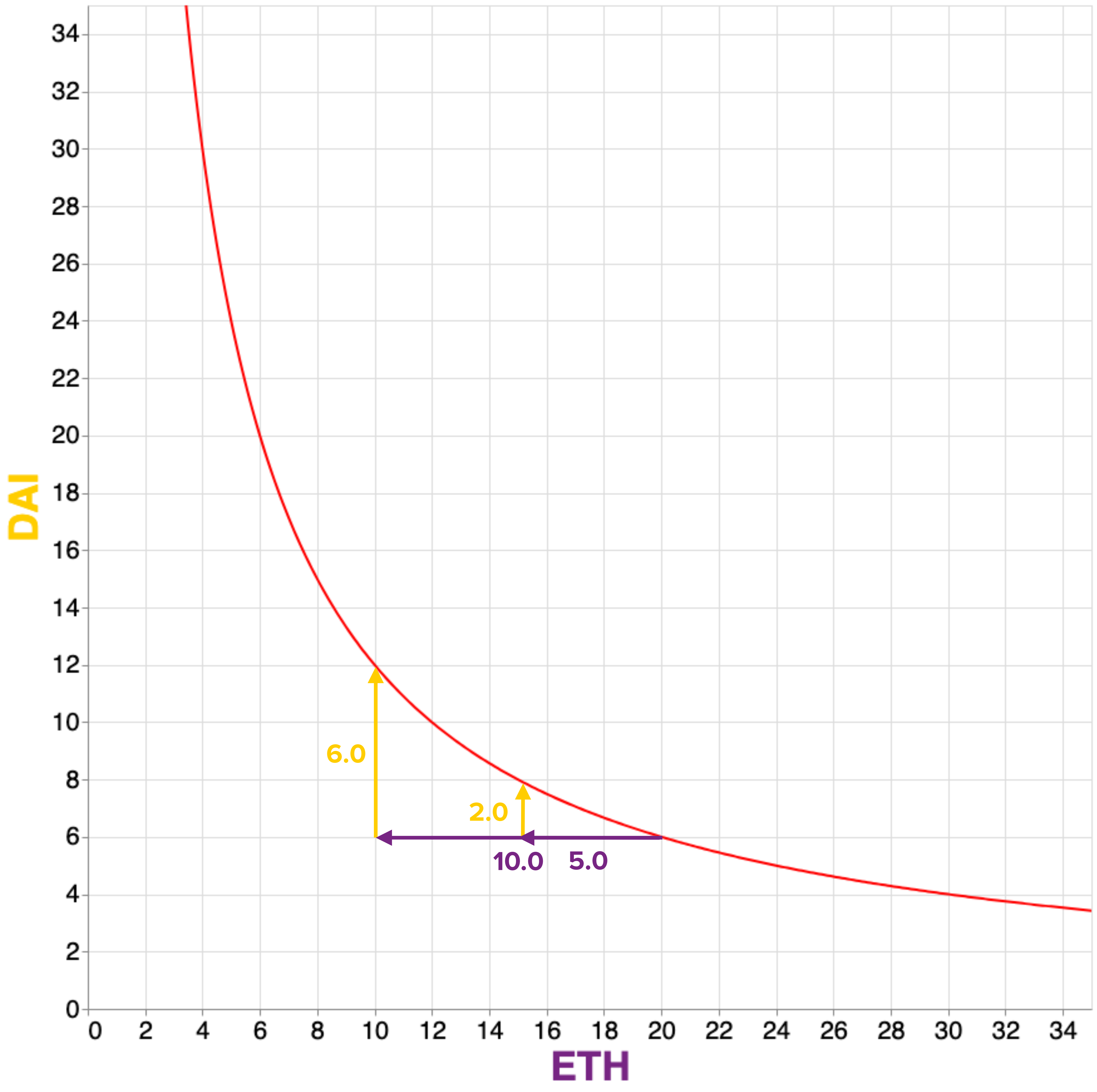
$$\Delta y = \frac{1}{\phi} \cdot \frac{y\Delta x}{x - \Delta x}$$

Divide both sides by $\Delta x$ to get $\Delta y/\Delta x$.

$$\boxed{\frac{\Delta y}{\Delta x} = \frac{y\phi}{x + \phi\Delta x}}$$

$$\boxed{\frac{\Delta y}{\Delta x} = \frac{1}{\phi} \cdot \frac{y}{x - \Delta x}}$$

$$xy = k$$




# Marginal Price & Slippage

**Selling $x$ for $y$**

$$\frac{\Delta y}{\Delta x} = \frac{y\phi}{x + \phi \Delta x}$$

**Buying $x$ for $y$**

$$\frac{\Delta y}{\Delta x} = \frac{1}{\phi} \cdot \frac{y}{x - \Delta x}$$

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Observation #1

Pricing depends on the size of the trade, $\Delta x$.

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For example with **20.0 ETH** * **6.0 DAI** = **120**,

Buying 10 ETH (i.e. $\Delta x = 10$) costs 6.02 DAI*

Or 0.602 DAI per ETH

Whereas buying 5 ETH costs 2.006 DAI

Or 0.401 DAI per ETH

* assuming $\phi = 0.997$

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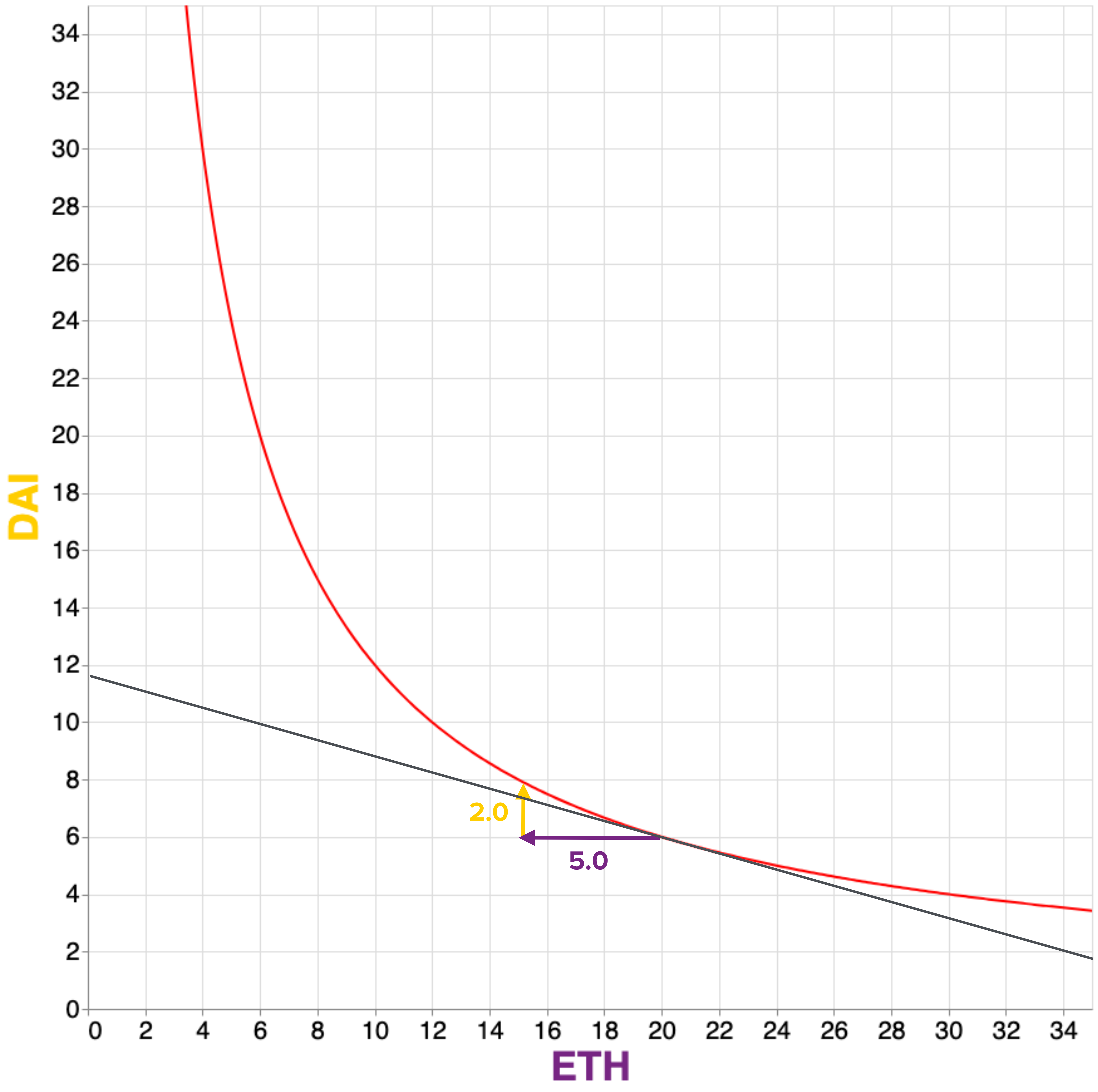
$$xy = k$$




# Marginal Price & Slippage

**Selling $x$ for $y$**

$$\frac{\Delta y}{\Delta x} = \frac{y\phi}{x + \phi \Delta x}$$

**Buying $x$ for $y$**

$$\frac{\Delta y}{\Delta x} = \frac{1}{\phi} \cdot \frac{y}{x - \Delta x}$$

In the limit, as $\Delta x$ approaches 0:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \phi \frac{y}{x} \qquad \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{1}{\phi} \frac{y}{x}$$

And, if we set the fee to zero ($\phi = 1$), then:

$$\boxed{M_p = \left| \frac{y}{x} \right|}$$

where $M_p$ denotes *marginal price*

$M_p$ is equal to the magnitude of the slope of the tangent line.

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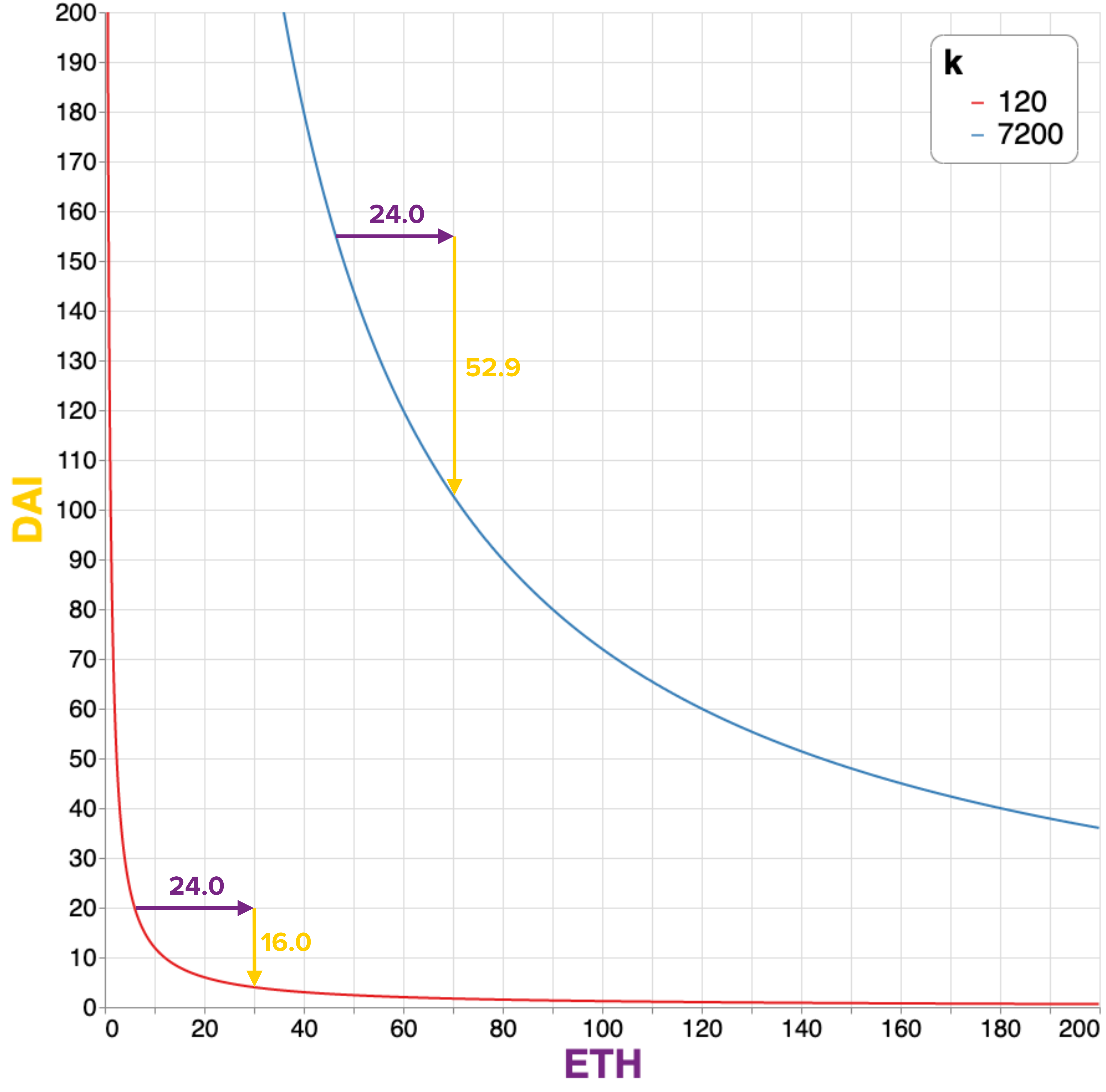
$$xy = k$$




## Marginal Price & Slippage

**Selling $x$ for $y$**

$$\frac{\Delta y}{\Delta x} = \frac{y\phi}{x + \phi \Delta x}$$

**Buying $x$ for $y$**

$$\frac{\Delta y}{\Delta x} = \frac{1}{\phi} \cdot \frac{y}{x - \Delta x}$$

---

Observation #2

Pricing depends on the size of $x$ and $y$ (i.e. $k$)

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It's straightforward to see that, as $k$ increases, the effective price of the AMM is less sensitive to $\Delta x$.

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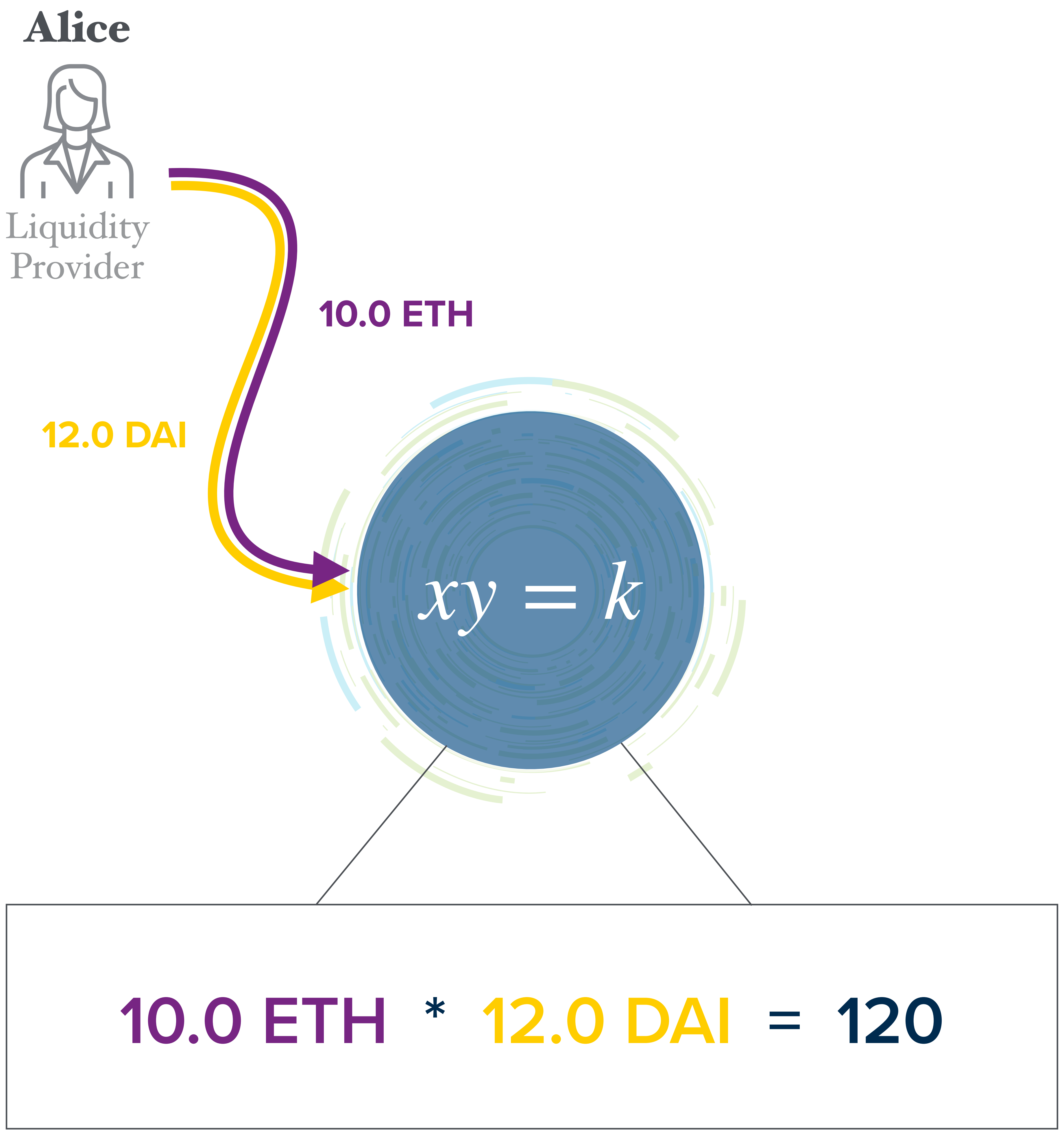
# Incentives for Liquidity Providers




Alice deposits 10 ETH and 12 DAI of liquidity, which implies:

$$M_p = 1.2 \qquad \text{where } M_p \text{ denotes } \textit{marginal price}$$

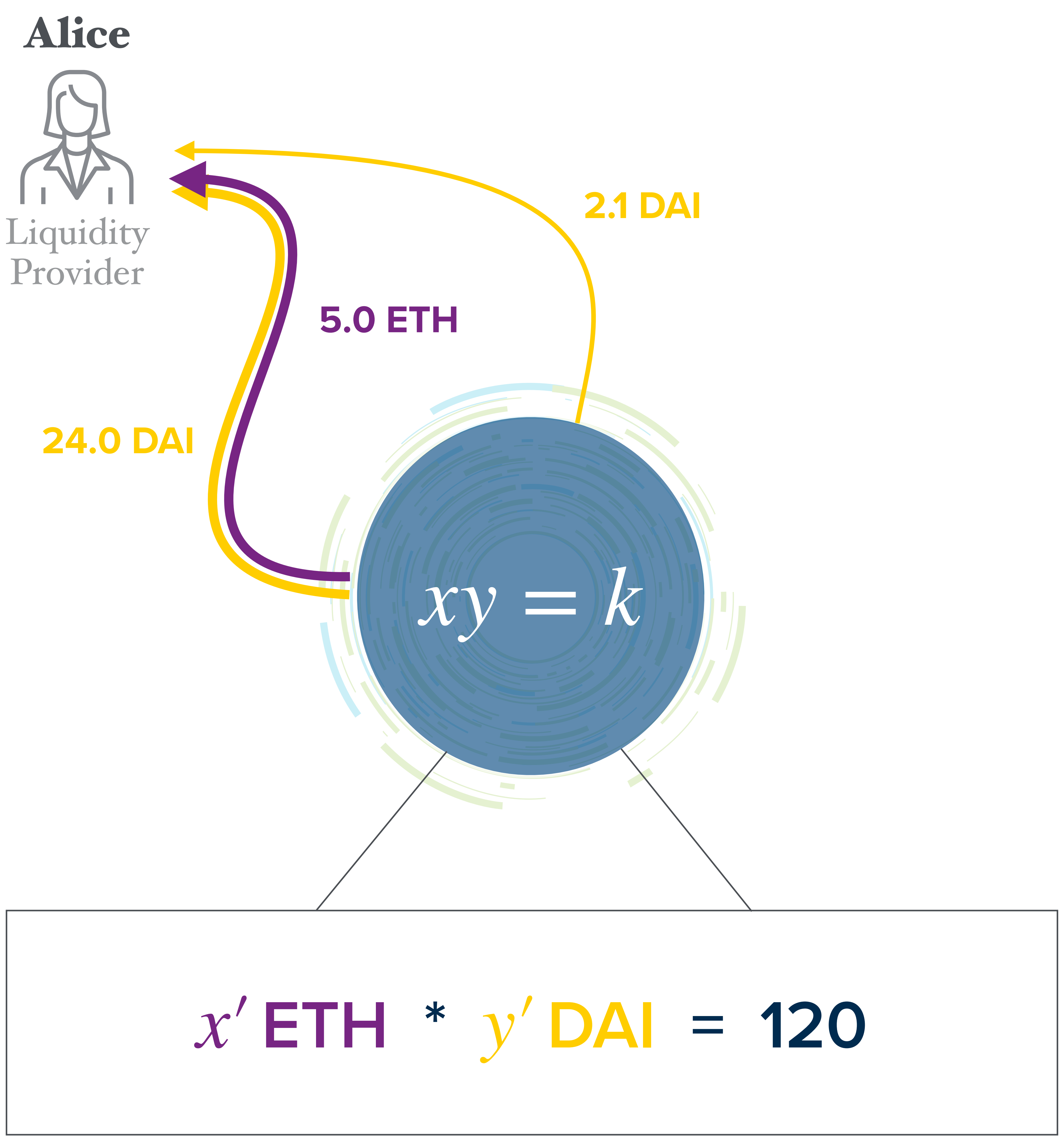
Alice waits for a month, during which traders drive $700 worth of volume through the AMM.

At the end of the month, Alice withdraws her ETH and DAI. By that time, the price of ETH has gone up 4x. The marginal price is now:

$$M_p' = 4.8$$

**What is Alice's return?**

Assume: $(1 - \phi) = 0.003$

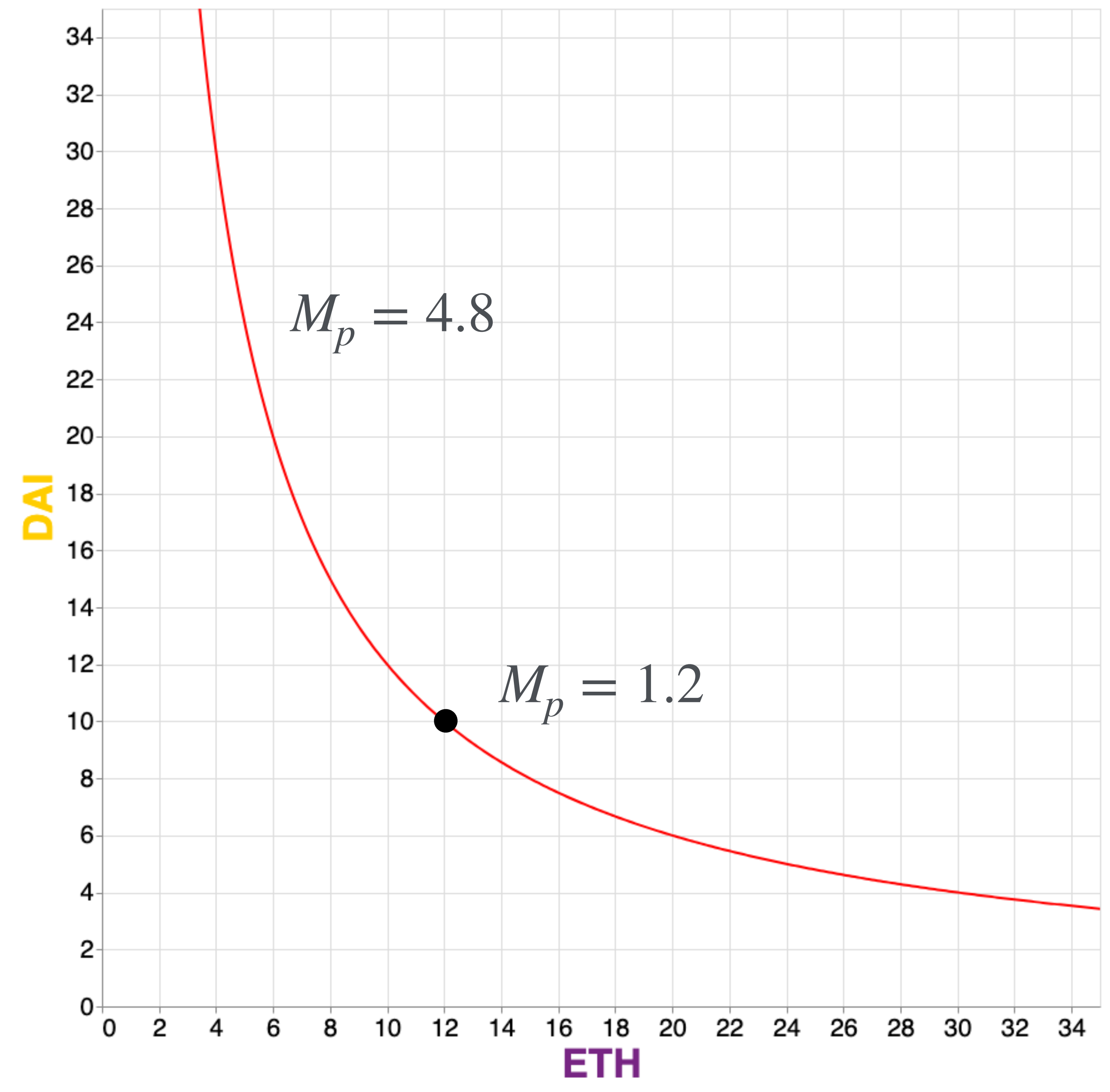



First, what does Alice earn from liquidity provider fees?

$$V(1-\phi) = 700 * 0.003 = \$2.1$$

where $V$ denotes trading volume

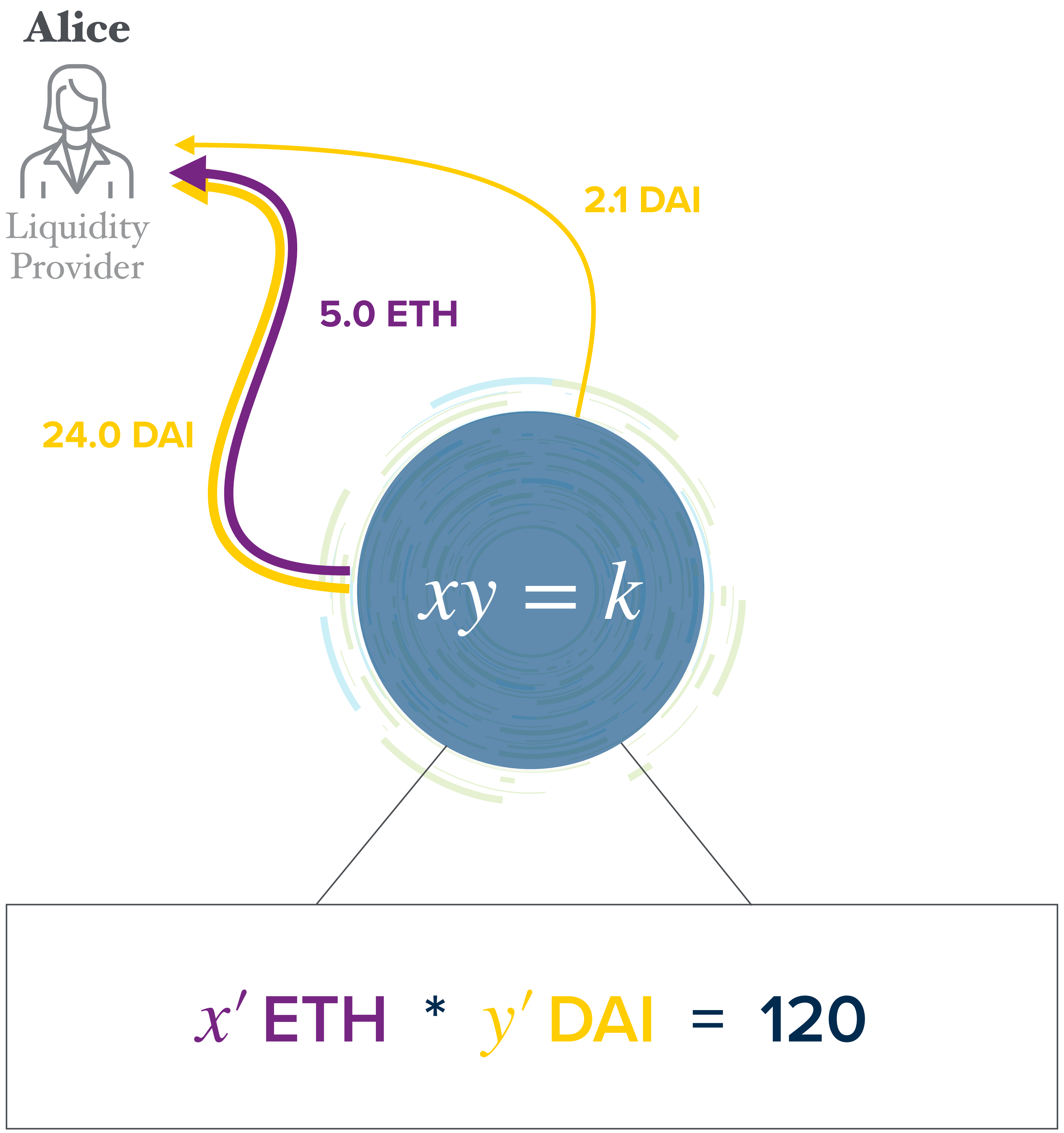
Second, how many ETH and DAI does Alice get back?




$x' = 5$ ETH

$y' = 24$ DAI

# ~~Impermanent~~ Divergence Loss




**So, how did Alice do?**

Measured in DAI, Alice now has:

$$R = 5\ \text{ETH} * \frac{4.8\ \text{DAI}}{\text{ETH}} + 24\ \text{DAI} + 2.1\ \text{DAI}$$

$$R = 50.1\ \text{DAI}$$

Not bad, but how would she have done if she had just held onto her 12 ETH and 10 DAI?

$$R_B = 12\ \text{ETH} * \frac{4.8\ \text{DAI}}{\text{ETH}} + 10\ \text{DAI}$$

$$R_B = 67.6\ \text{DAI}$$

**This is called ~~impermanent~~ loss**
**divergence**

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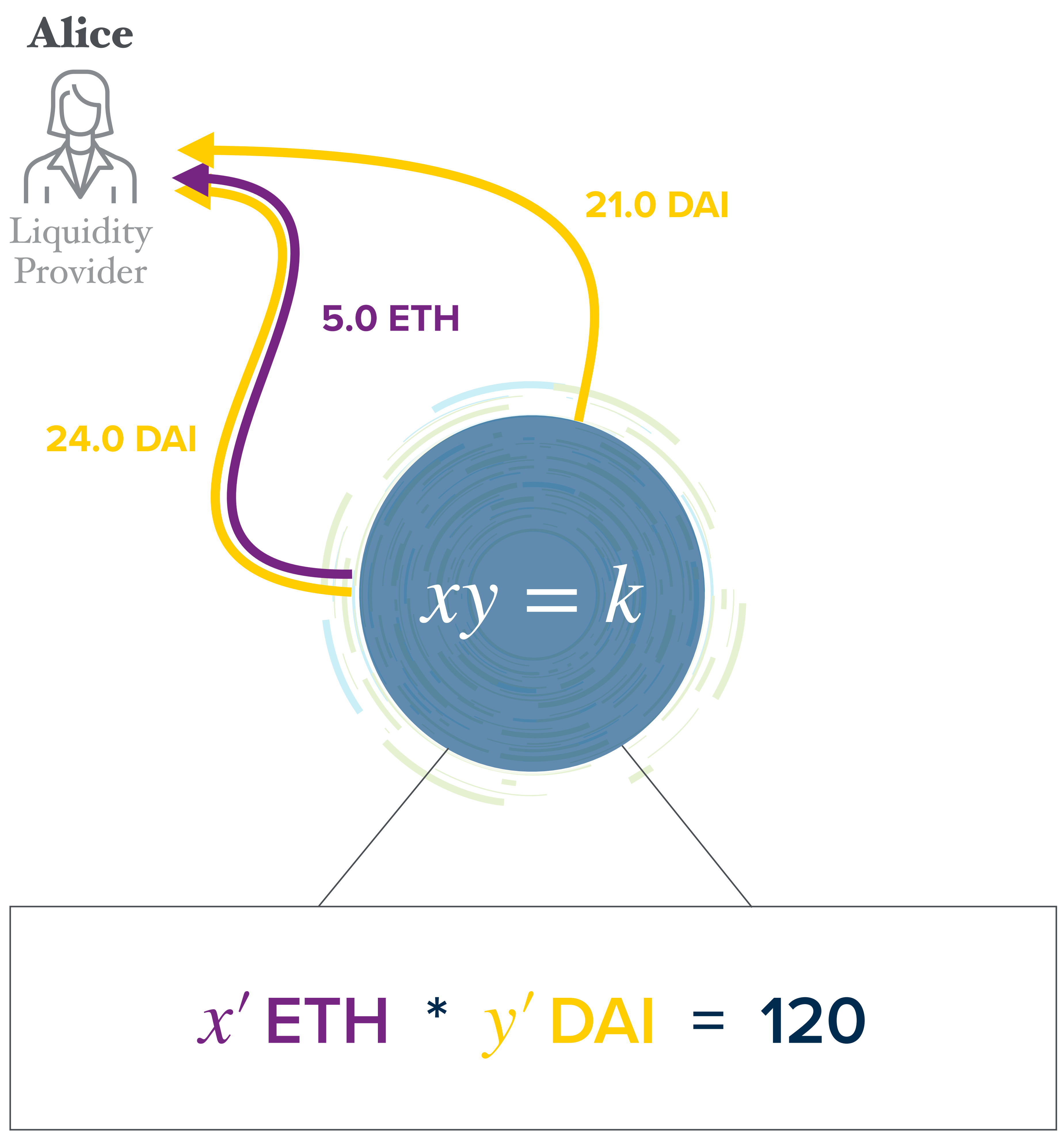
# ~~Impermanent~~ Divergence Loss




**What if volume had been higher?**

Say, volume had been $7,000 instead of $700:

$$V(1-\phi) = 7000 * 0.003 = \$21$$

Therefore,

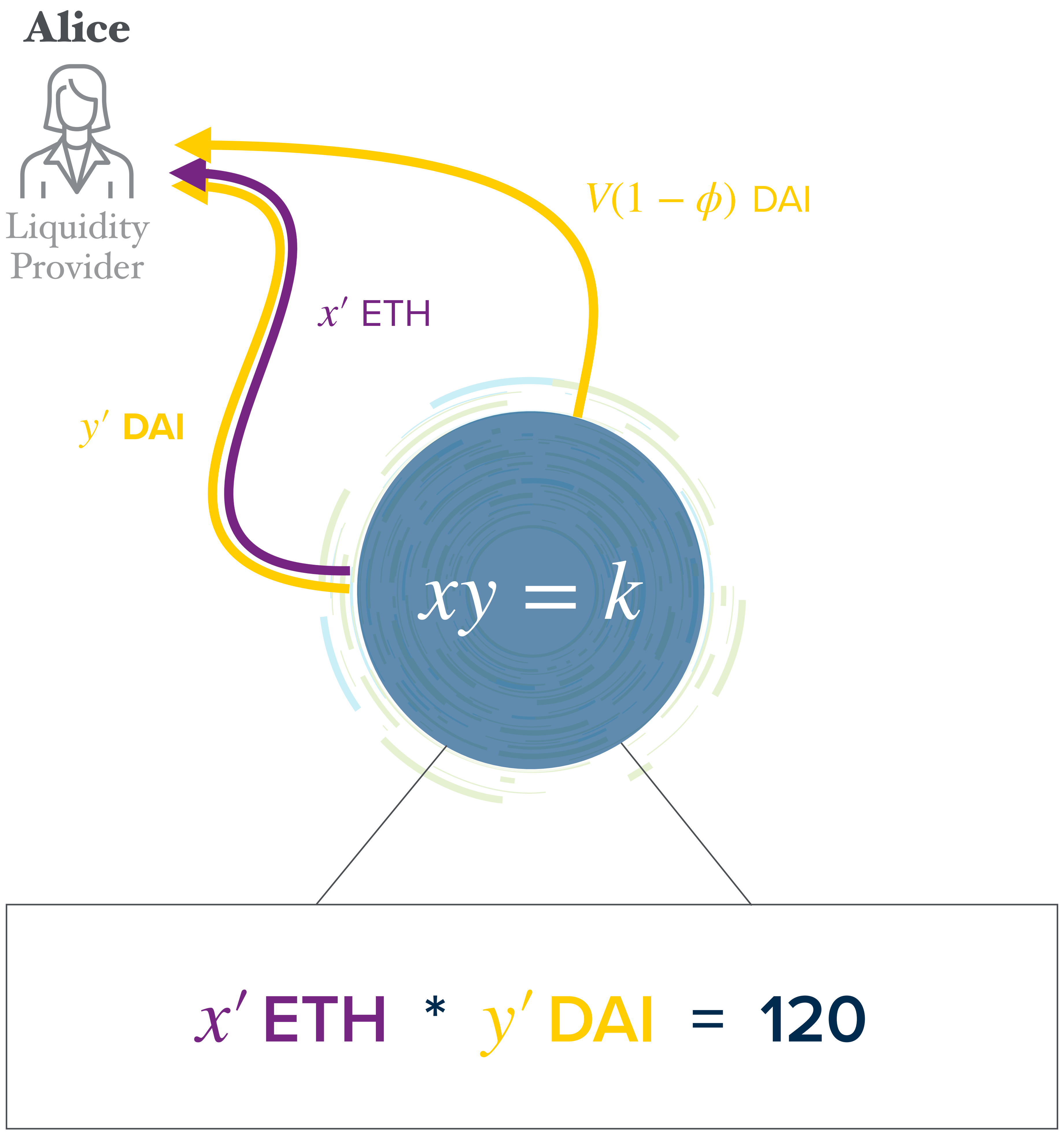
$$R = 5 \text{ ETH} * \frac{4.8 \text{ DAI}}{\text{ETH}} + 24 \text{ DAI} + 21 \text{ DAI}$$

$$R = 69.0 \text{ DAI}$$

This time, Alice's returns are greater than her baseline return $R_B$ of 67.6 DAI. Her profit:

$$P_L = \frac{R}{R_B} - 1 = 2.1\,\%$$

# ~~Impermanent~~ Divergence Loss




**More generally**

Alice's return $R$ is given by:

$$R = x'M_p' + y' + V(1-\phi)$$

Her baseline return $R_B$ is given by:

$$R_B = xM_p' + y$$

Her profit, in percentage terms is given by:

$$P_L = \frac{R}{R_B} - 1 = \frac{x'M_p' + y' + V(1-\phi)}{xM_p' + y} - 1$$

Let's ignore the volume term for now, and simplify:

$$P_L = \frac{x'M_p' + y'}{xM_p' + y} - 1 \quad \text{assuming } V = 0 \text{ for now}$$

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**Recall**

$xy = k$ and $M_p = y/x$

Thus,

$x = \sqrt{\frac{k}{M_p}}$ and $y = \sqrt{kM_p}$

Also,

$x' = \sqrt{\frac{k}{M'_p}}$ and $y' = \sqrt{kM'_p}$

Finally, let:

$M'_p = rM_p$

# ~~Impermanent~~ Divergence Loss

## Simplifying

$$P_L = \frac{x'M'_p + y'}{xM_p + y}$$

Step 1: let's express everything in terms of $M_p$ and $k$.

$$P_L = \frac{\sqrt{\frac{k}{rM_p}}rM_p + \sqrt{krM_p}}{\sqrt{\frac{k}{M_p}}rM_p + \sqrt{kM_p}} - 1 = \frac{2\sqrt{r}\cancel{\sqrt{kM_p}}}{r\cancel{\sqrt{kM_p}} + \cancel{\sqrt{kM_p}}} - 1$$

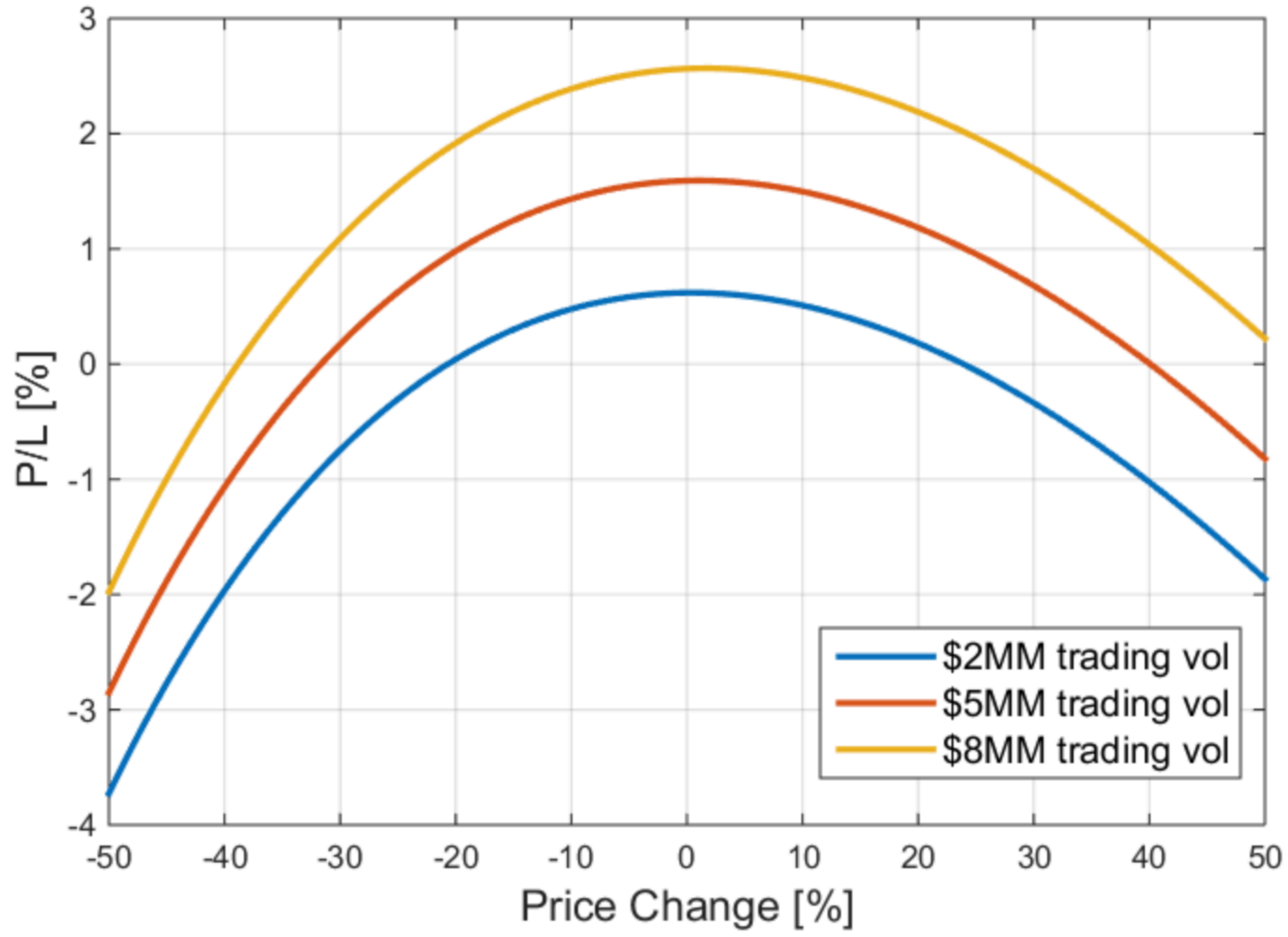
$$P_L = \frac{2\sqrt{r}}{r+1} - 1$$

Step 2: Reintroduce the volume term:

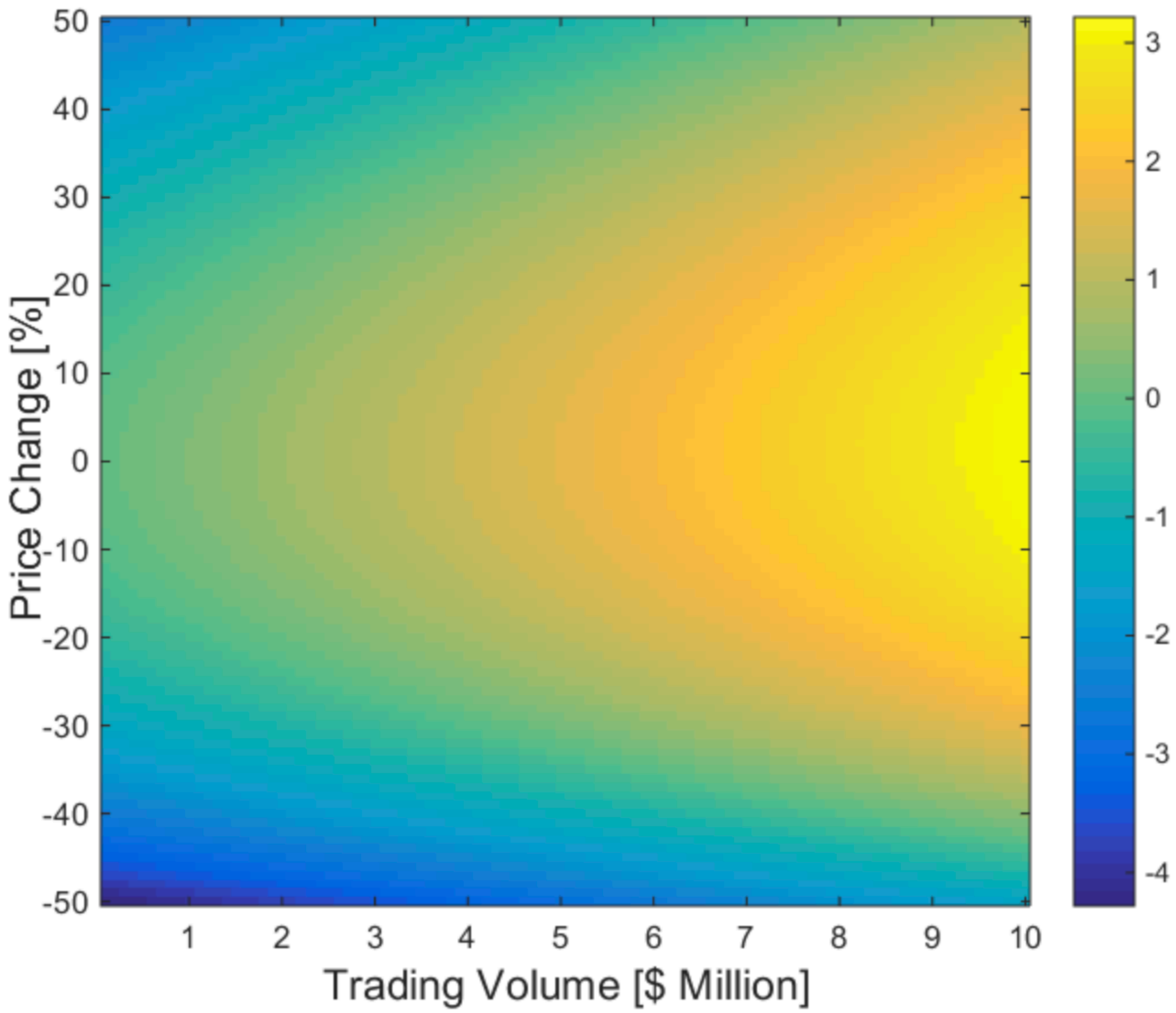
$$P_L = \frac{2\sqrt{r}}{r+1} + \frac{V(1-\phi)}{c} - 1$$

Step 3: Plot this equation

# ~~Impermanent~~ Divergence Loss




Optimal P/L occurs when the final price is equal to that at liquidity provisioning




P/L percentage of liquidity provision on Uniswap for different scenarios of exchange trading volume and ETH price change

$$P_L = \frac{2\sqrt{r}}{r+1} + \frac{V(1-\phi)}{c} - 1$$

Image credit: https://www.tokendaily.co/blog/pnl-analysis-of-uniswap-market-making

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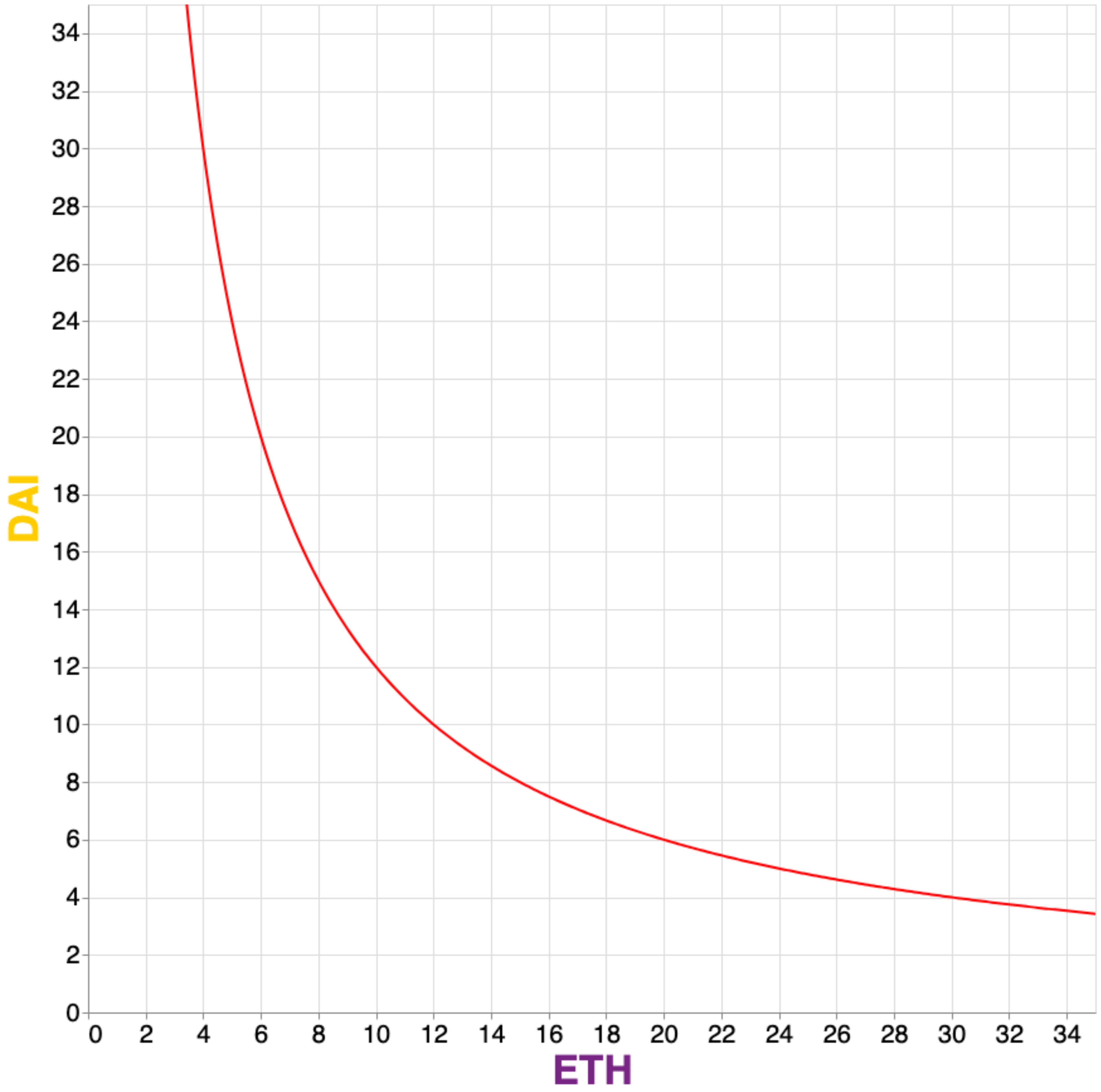
# Uniswap ROI By Token

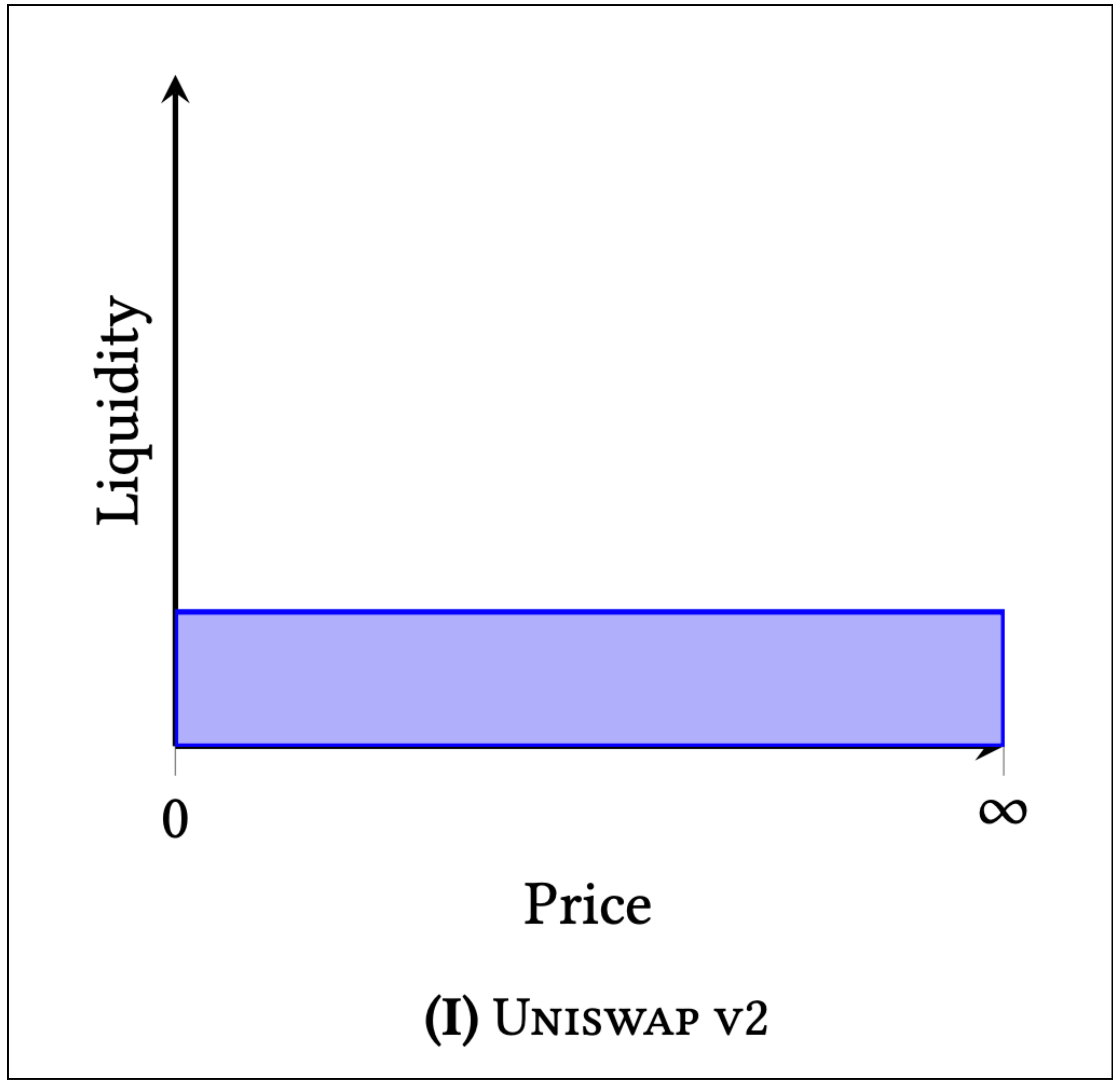
Quick Demo: https://zumzoom.github.io/analytics/uniswap/roi/

# Big Limitation of Uniswap V2

## Capital Efficiency

# Distribution of Liquidity

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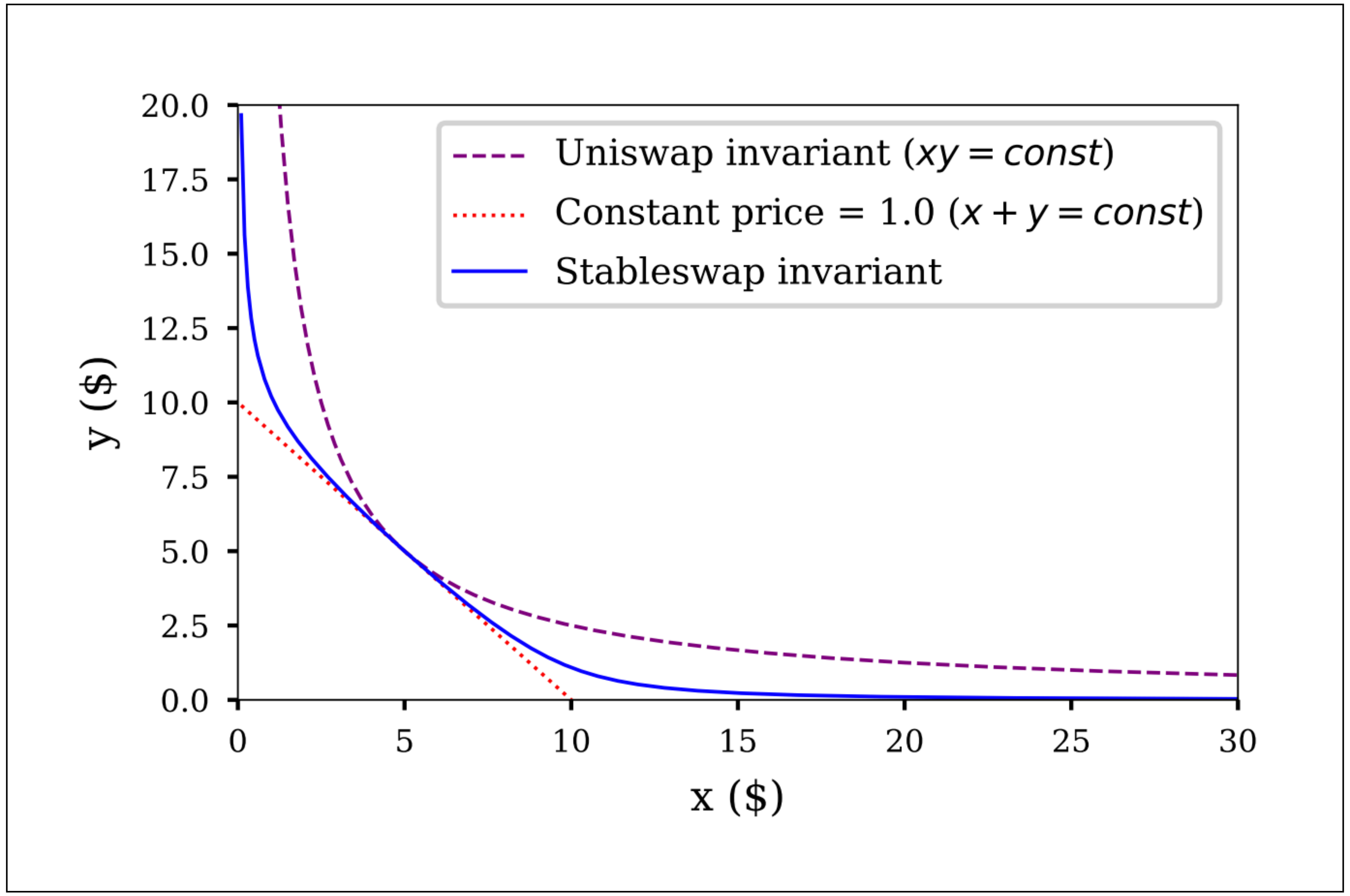
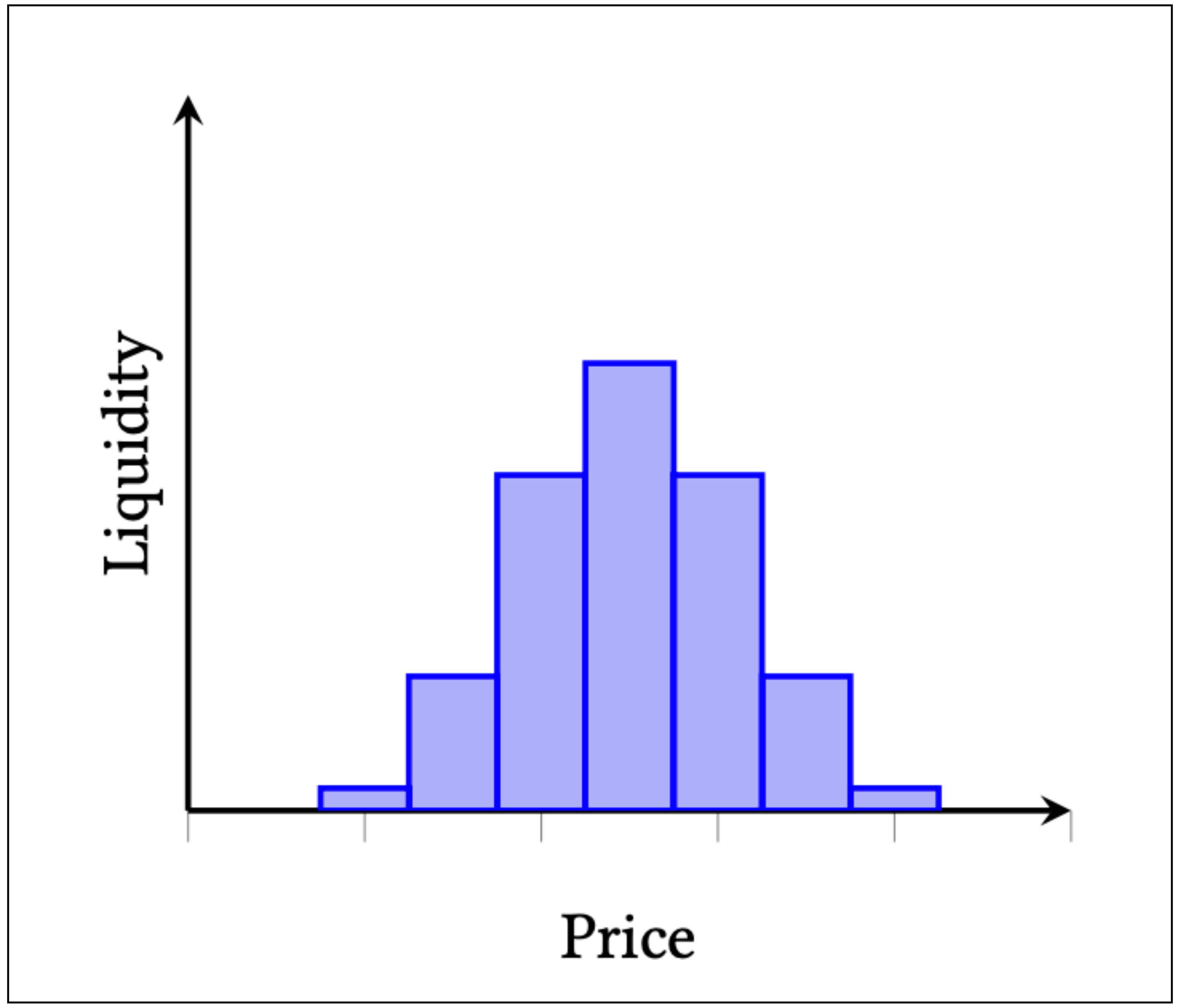
# One Approach: Curve.Fi



# Uniswap V3: Universal AMM

Demo: app.uniswap.org

# Concentrate Liquidity

https://uniswap.org/blog/uniswap-v3/

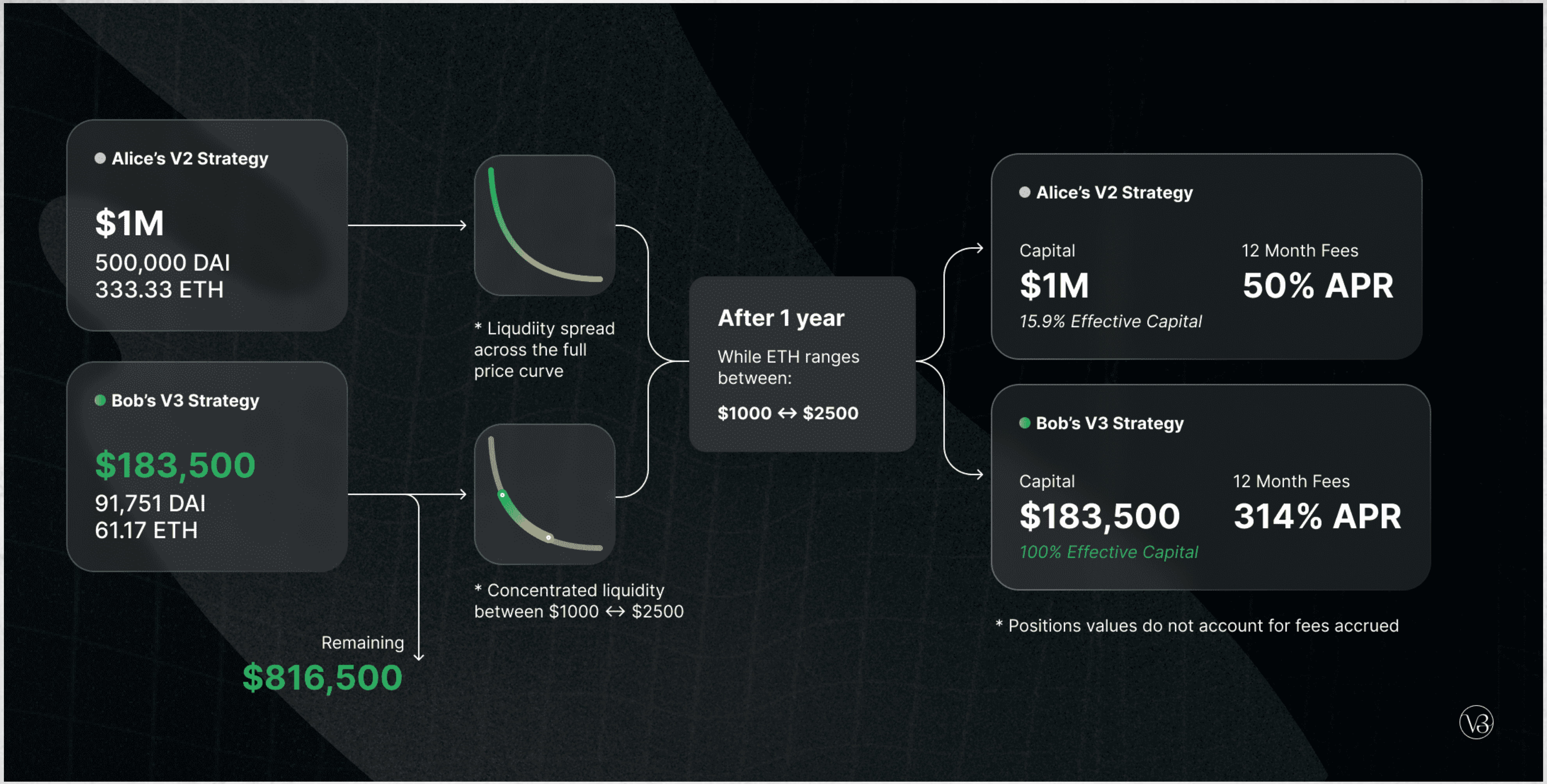
# Narrow Activation

https://uniswap.org/blog/uniswap-v3/

# Unified Pool

https://uniswap.org/blog/uniswap-v3/

# Capital Efficiency: Example




https://uniswap.org/blog/uniswap-v3/

# White Paper

## Uniswap v3 Core

March 2021

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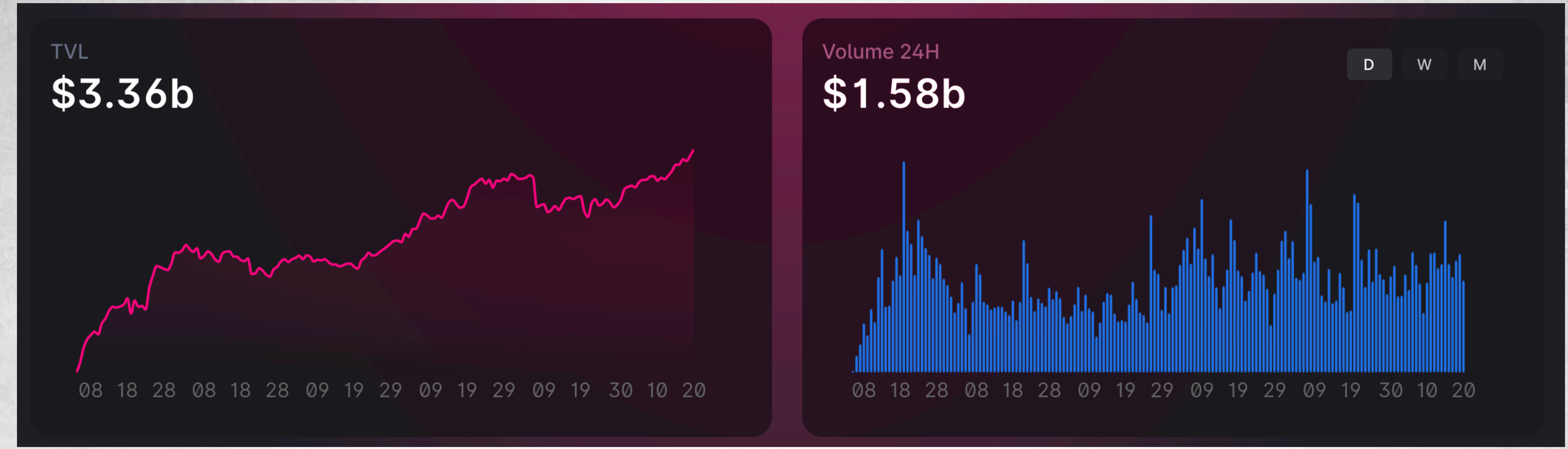
https://uniswap.org/whitepaper-v3.pdf

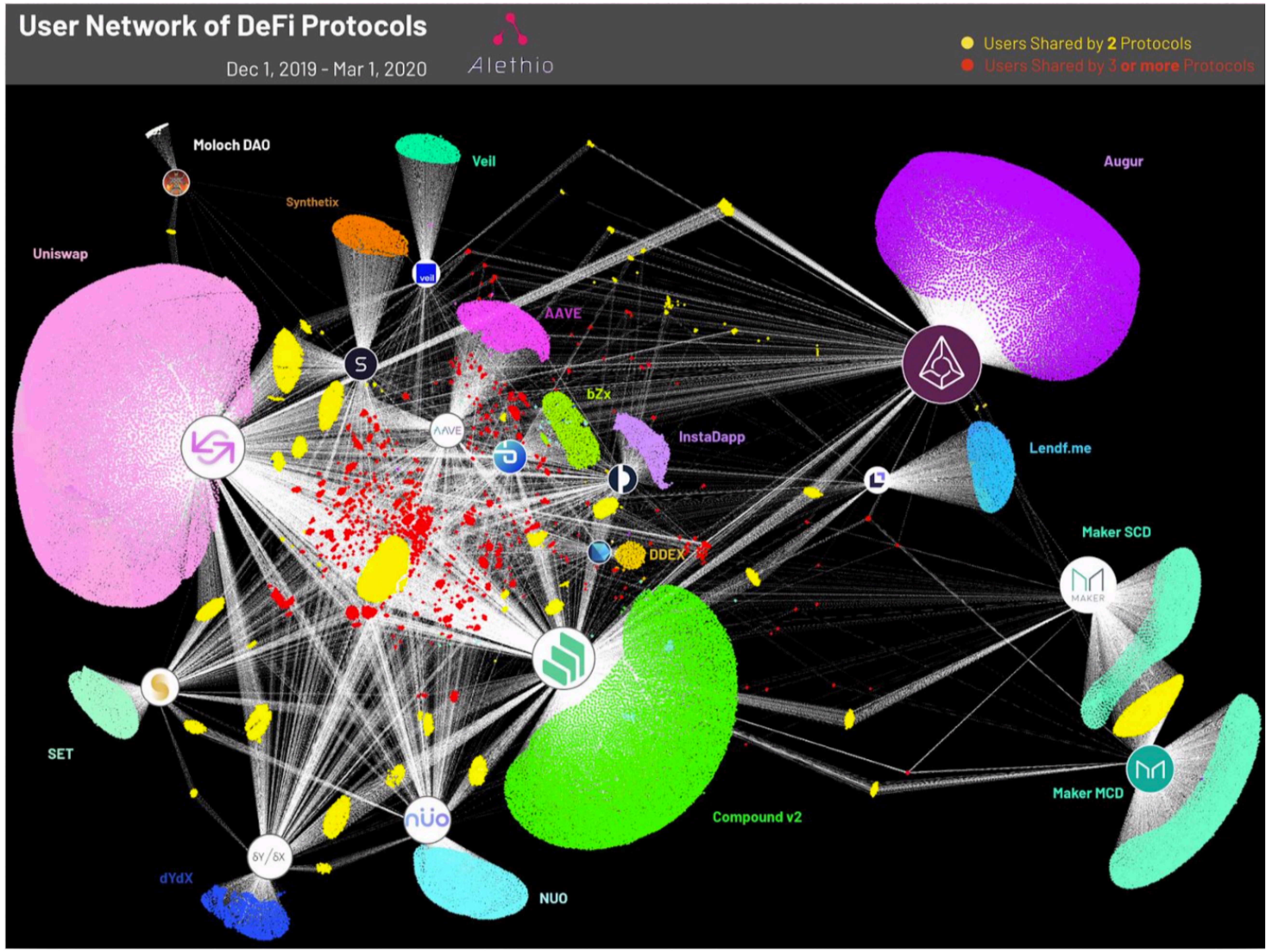
# Uniswap's Metrics To Date




Quick Demo: https://uniswap.info/

# Example: Uniswap

## Interoperability



# DEXs: Concluding Thoughts

## Desired Characteristics

- Simple — buildable as a smart contract
- Automated liquidity — no dependence on active market-makers
- No single points of control — no dependence on centralized parties
- Composable/Programmable