CS251 Fall 2020

(cs251.stanford.edu)

Randomness Beacons and VDFs

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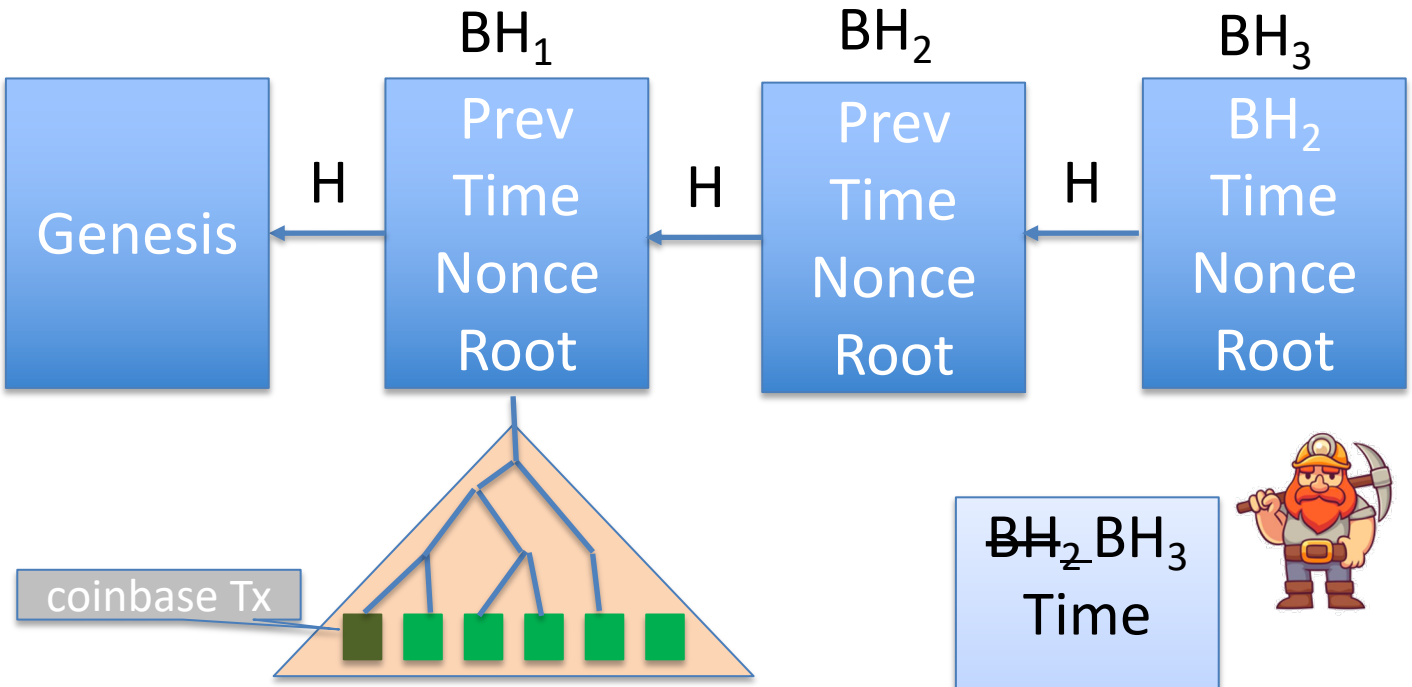
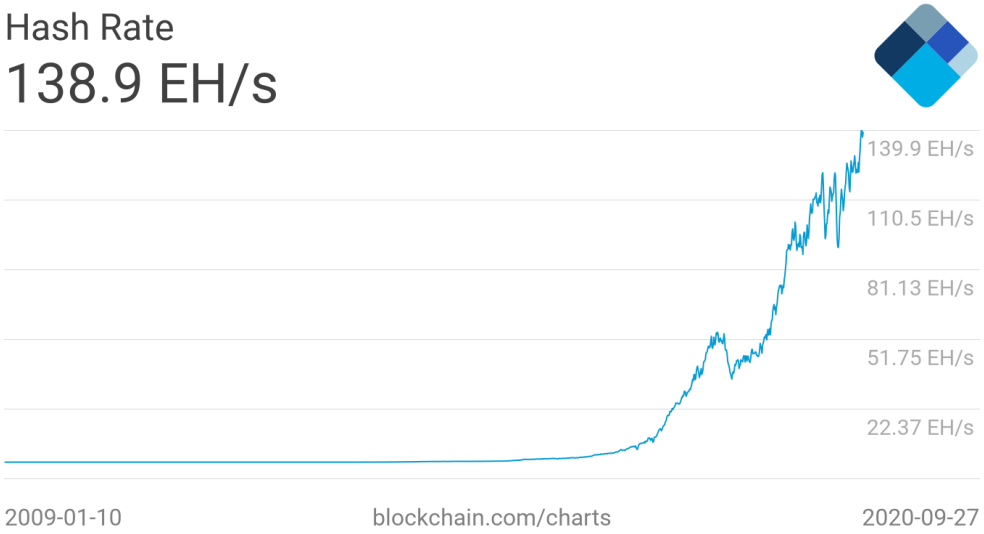
Recap: Nakamoto Consensus



Nakamoto Properties

- Anonymous participation
- Nodes can join/leave
  - Very scalable
  - Sleeping Beauty property
- Leader not known beforehand
  - Makes bribing harder
- Up to ½ corruptions
- Slow
  - Even when everyone is honest
- Resource intensive
  - PoS based possible
- Long forks possible
- No guarantees under long delays

Difficulty Resets



- Computation increases
- But block time ~constant

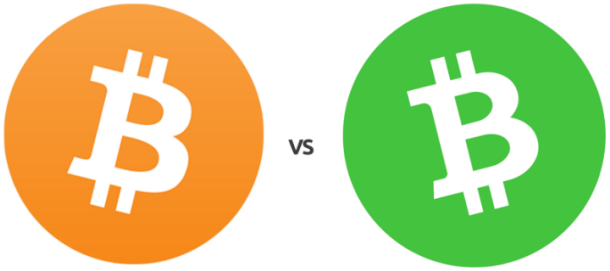
- Every two weeks difficulty reset based on prior two weeks
- Based on time stamps
- Slightly lagging
- Miners accept *heaviest* chain

Difficulty Reset Attacks

- Attacker with 1/3 hash power, Difficulty 1
- Fork 100 blocks deep
- Modifies time stamps on private fork such that blocks look like they are mined in short succession
- Increases difficulty to 200
- Probability that attacker will mine 1 block of difficulty 200 while honest chain produces 100 blocks of difficulty 1:
  - Poisson distribution with expectation $1/6^{th}$
  - $\Pr\left[X \geq 1, X \sim Poisson\left[\frac{1}{6}\right]\right] = 15.3\%$
- Defense: Max difficulty change 4x, $1/4^{th}$ (Magic number)

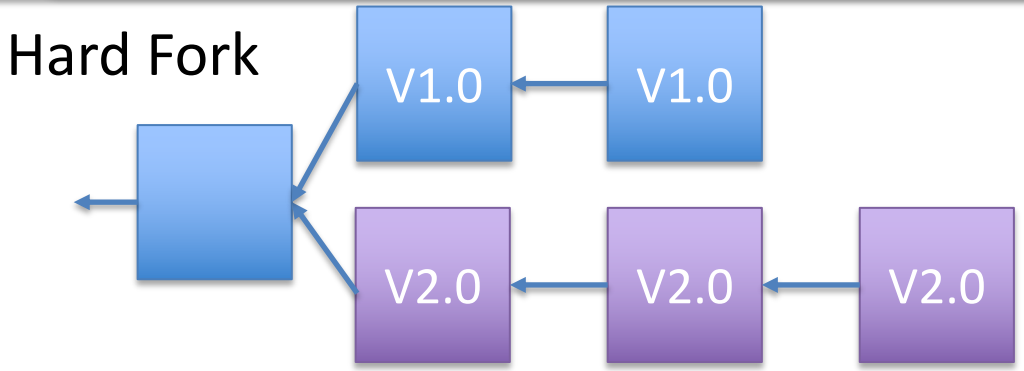
Changing the rules/Governance

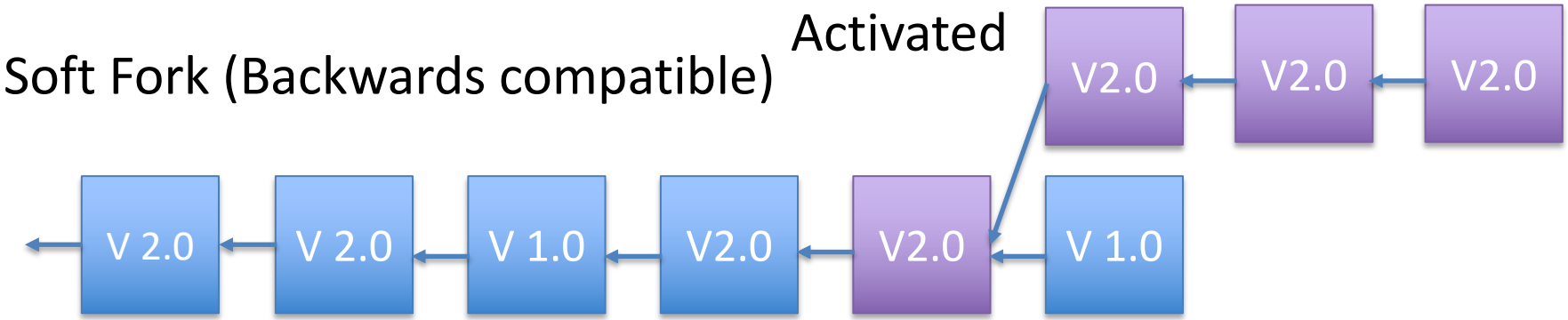
- Protocol upgrades
  - New Transaction types (Add Smart Contracts)
  - New Consensus (Switch from PoW to PoS)
  - Increase Blocksize (1MB) Bitcoin/Bitcoin Cash



- How do we reac se things

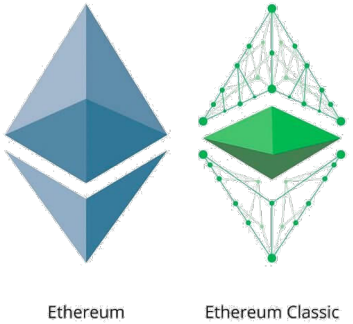
Soft/Hard Fork Activiation





Hard Forks

- Technically the simplest
- New protocol version (new software)
- Everyone upgrades
- New protocol incompatible with old protocol
- Everyone needs to upgrade
- Ethereum/Zcash/Monero do this semi regularly
- *Contentious* Hard Fork: Both versions exists
  - Need to worry about replay attacks

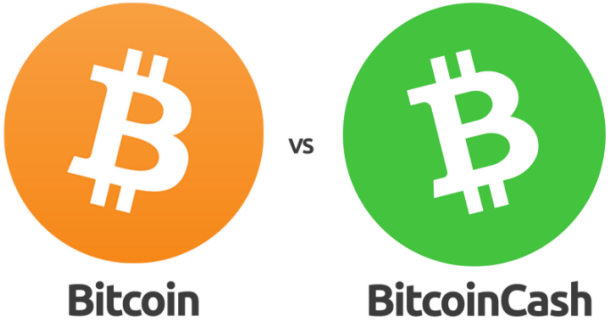


Soft Forks

- Rules become more restrictive
- Disabling old OP_CODES
- Further specifying signatures (ECDSA)
- Old clients still work but their transactions may get rejected
- If >50% upgrade then new rules enforced
- Segregated Witness was a contentious soft fork

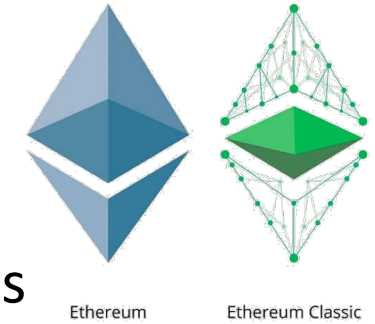
Case Study: Bitcoin vs Bitcoin Cash

- Bitcoin Blocks are limited to 1MB
- ~Roughly 7 tx/s
- Proposal to increase block size
- Opinion 1: “Larger blocks increase network delay, decreases security, transactions should be moved off the chain.”
- Opinion 2: “Bitcoin can support more transactions we should increase block size.”
- Split in 2017: Every Bitcoin user got same amount of Bitcoin Cash (sum is less than sum of parts).

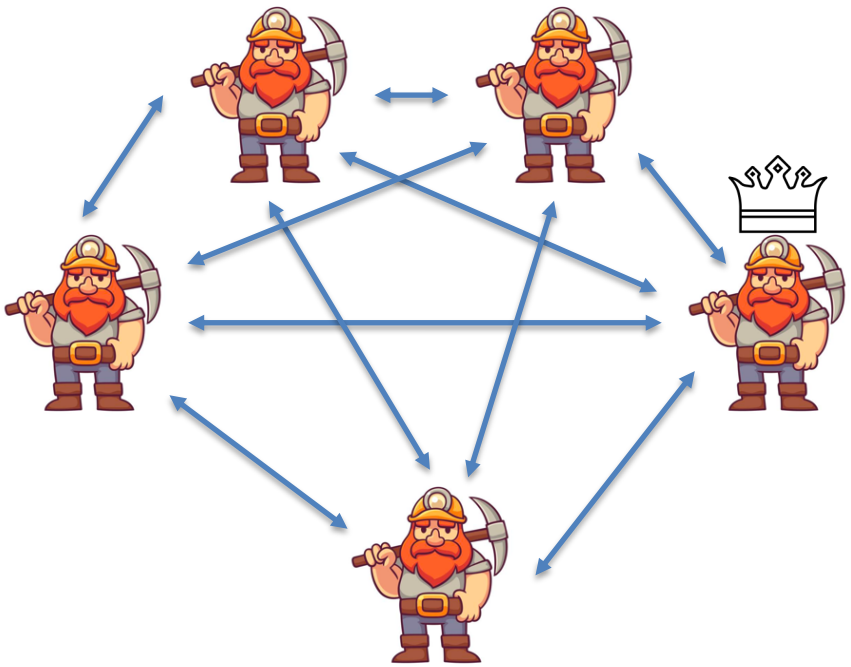


Case Study: Ethereum vs. Classic

- Ethereum had a smart contract called *DAO*
- Smart contract had a bug
- July 2016, $50 Million USD of Ether stolen
- Proposal to hard fork Ethereum and return funds
- Stake vote was held
  - 87% in favor but only 5.5% participated
  - 4 days later Ethereum forked
  - “Classic” is the old version including stolen funds
- Ethereum Foundation owns trademark and branded Fork Ethereum
- Later more divergence: Ethereum will move to PoS, Classic stay on PoW



Recap Byzantine Consensus



- Fast
- Partially Synchronous
- No wasted energy

- Known committee
  - (must communicate)
- Large committee
  - Large communication
- Predictable Leader
  - Bribing

Proof of Stake

Replace Sybill resistance of PoW with money

Stakes coins (through transaction)

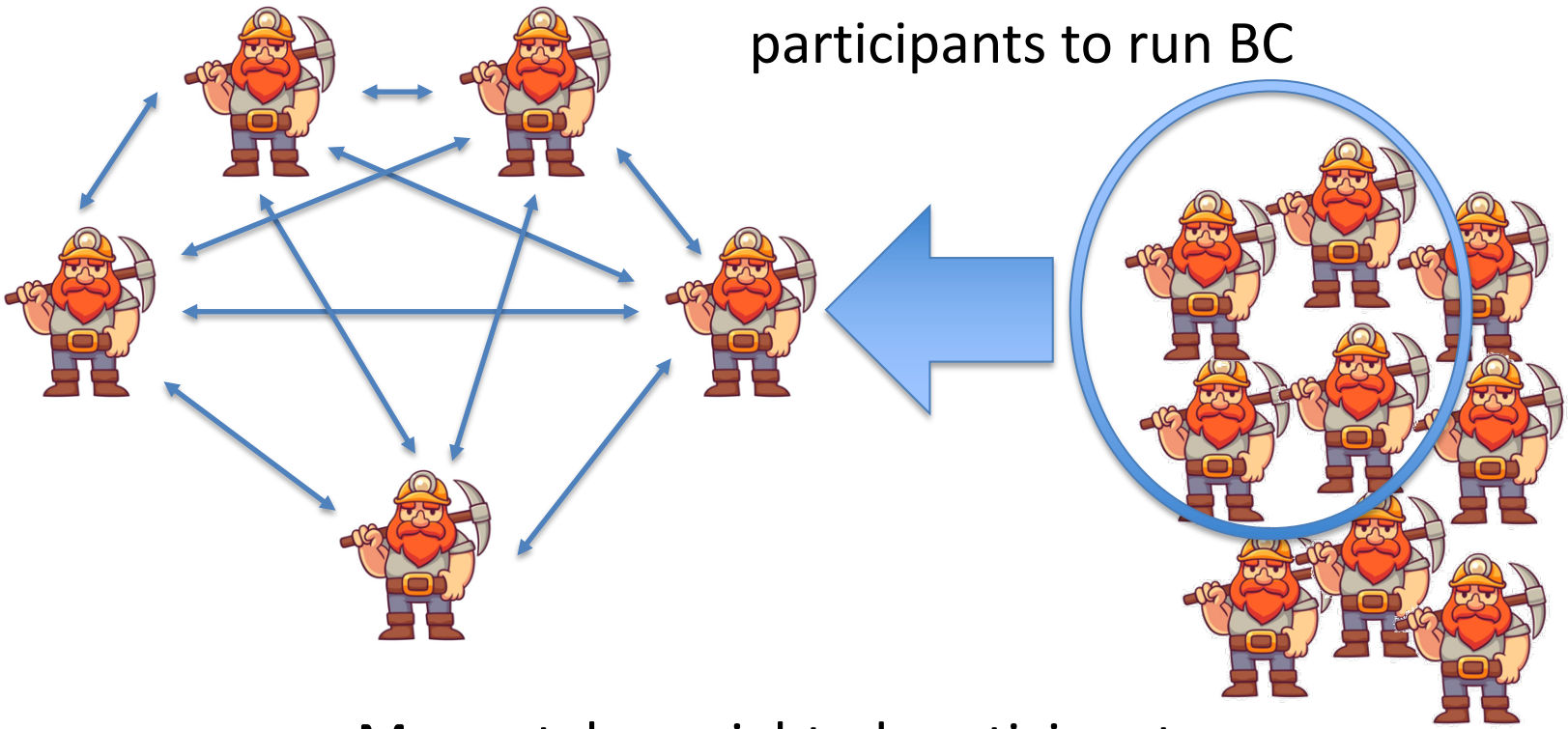
Can’t use staked coins for anything else!

Incentives: Get’s rewards/fees. Can use punishments/slashing

Voting Power: Proportional to relative stake

Scaling Byzantine Consensus

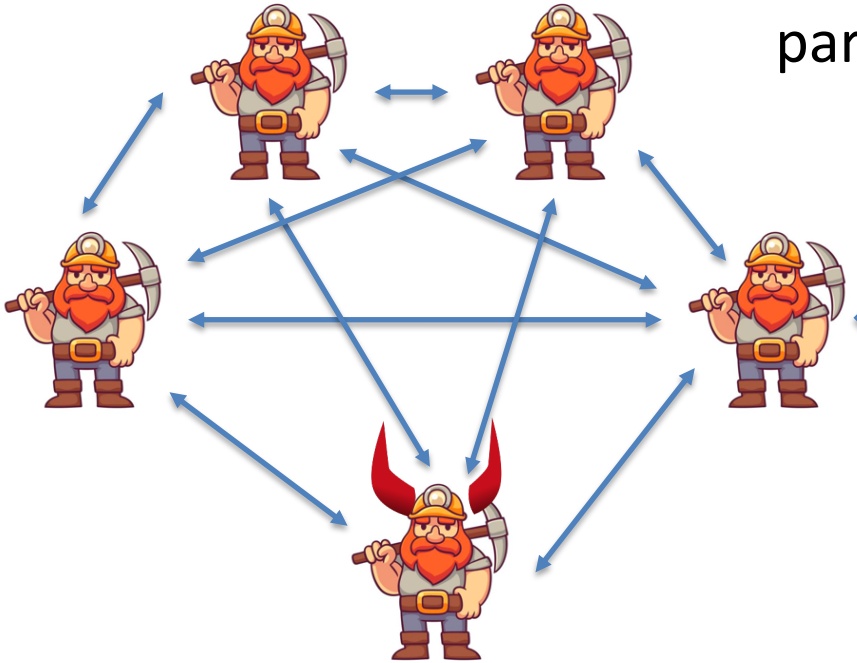
Sub select a set of participants to run BC

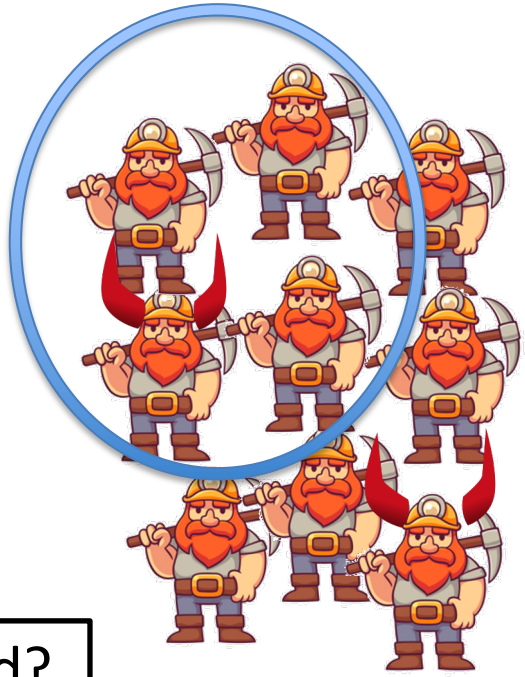


Many stake weighted participants

Scaling Byzantine Consensus

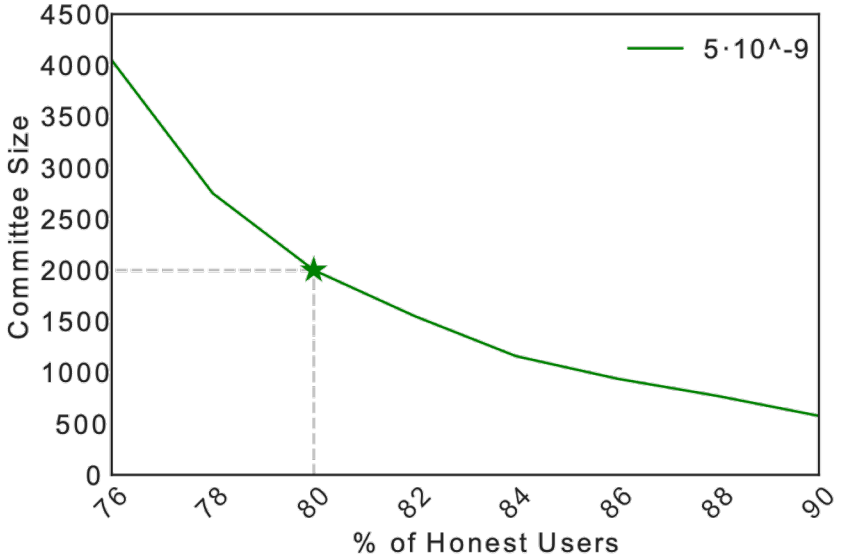
Sub select a set of participants to run BC

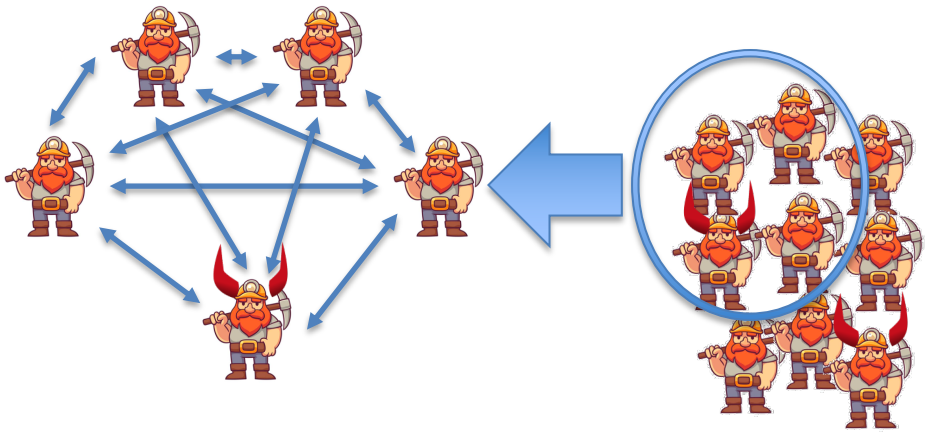




What fraction of committee will be corrupted?

Scaling Byzantine Consensus





100s of nodes
>67% Honest

>1000s of nodes
80% Honest

Random Selection

How to choose committee?

Proposal:

- Each staker computes H(block number,PK)
- If H(block number,PK)< target
  - Become part of committee for round
- If BC succeeds add Block to chain
- Target such that ~1000 nodes win

Broken! Attacker can choose PK such that they win

Randomness beacon

*An ideal service that regularly publishes random value which no party can **predict** or **manipulate***

01010001 01101011 10101000 11110000

Random Selection with Beacon

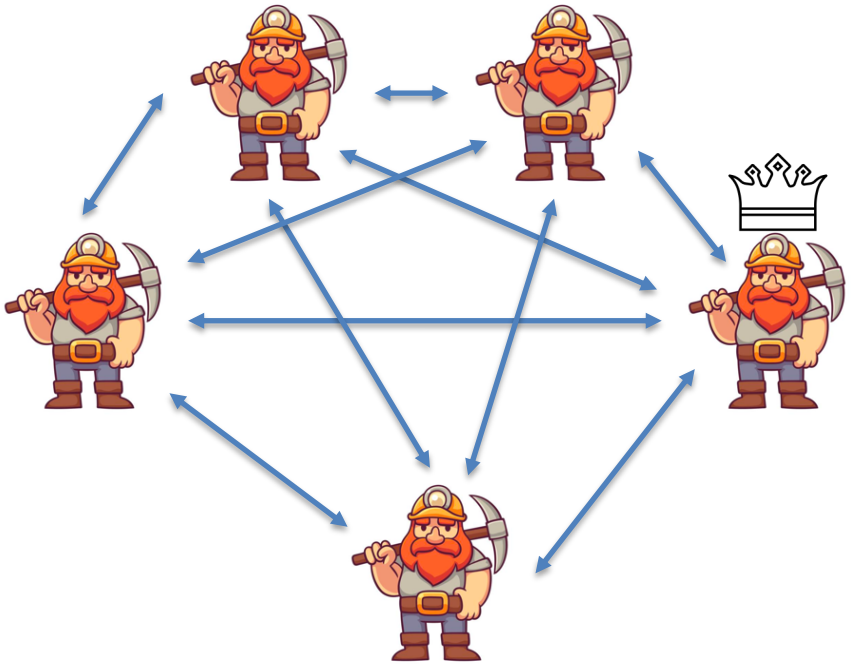
How to choose committee?

- Each Block wait for beacon randomness
- Each staker computes H(~~block number~~ beacon, PK)
- If H(~~block number~~ beacon,PK)< target
  - Become part of committee for round
- If BC succeeds add Block to chain

Beacon unpredictable so can't choose PK

Even better: Compute deterministic (BLS) signature on Beacon and use as ticket (prevents others from seeing who won) VRF

Leader Selection



We can also make leader election random with a beacon!

Can make BC resilient vs. adversary that corrupts *adaptively* (Bribing)

See Algorand reading

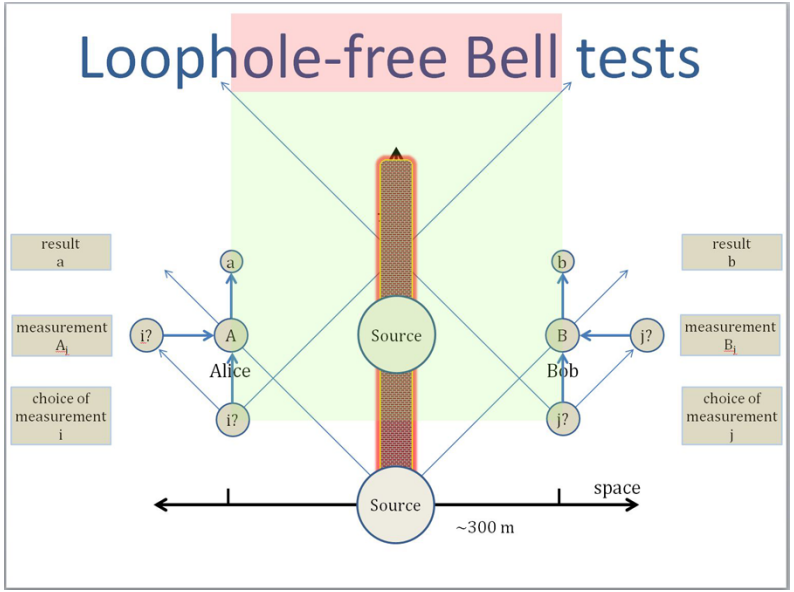
Lotteries

``Public displays'' can be corrupted

A beacon can be used to run a fair lottery

How to build a Beacon?

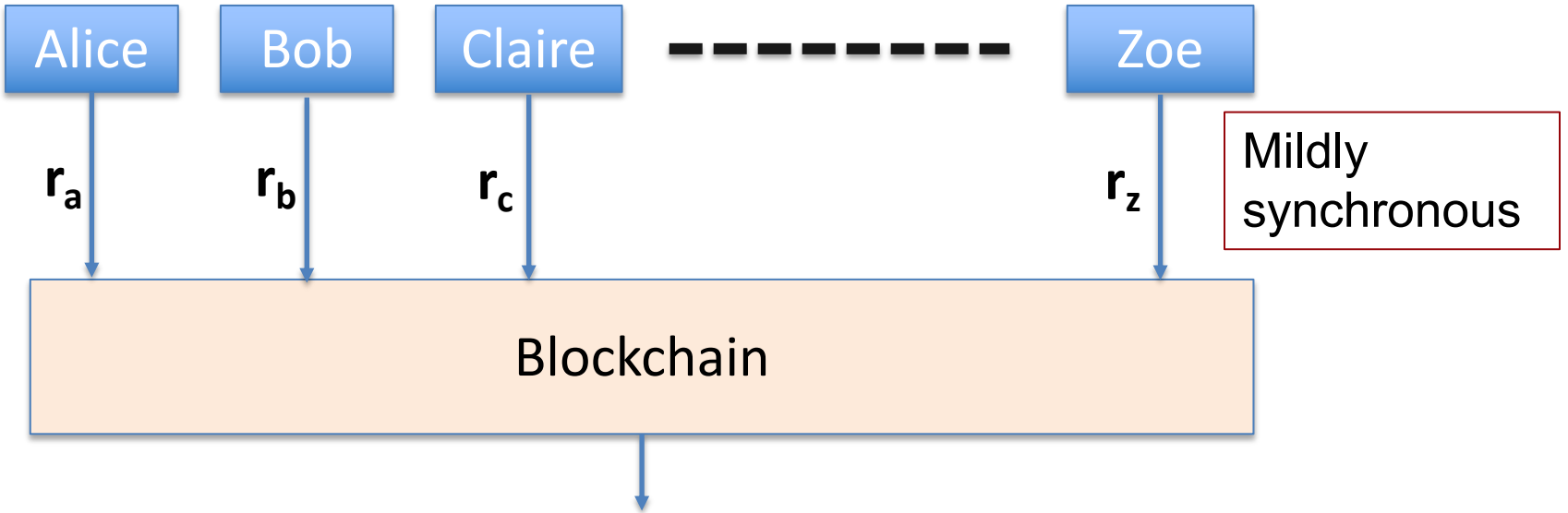
NIST (NSA) Beacon



Beacon Record

Version:	Version 1.0
Frequency:	60 seconds
Time:	08/13/2014 12:36 pm (1407947760)
Seed Value:	27D7280A657B5E0A99721D47E21A2276C80B5CDFDCA605E397D8BBAA51C24A06 40CC9C6EEB83BBB3D837011CA5B6CA08FADC78E2B8D36C75CC971757F82068A4
Previous Output:	2F2DE0662028D3C4D6F8DD7936262D9AFBDCFD0BD14BC733E257B14F48881A99 206BBC9429FD9BFE719551EAB840CEE8157ACAEBC80342CE4B66443C0859E216
Signature:	986C73CF88056635C5E0A018358D0D91CF10A2F2B16C8B8D91AA34B0A04D103B CFF347B714DAC343D5838E07FFDFC49BE6E39811350DC0193D17CFE1BC4EDB5B 7E3AC425EF7840EF4E549D66D0F0FB383DD9F29DFDAEF2E520B8606A4F6C55FB 3B766CC9D66494FAC1FE8983D58525224778F5AE3C3727FF0AC71DCE3B30E33B A6CFD767EE3D299A5324E371AFB49AEC46F88D6DCAE6FCBF8B93D461B84C59CB 7577BE9A63FE0DB7C83944B545C501A4C787F87B15A0F8CFD8FB7FC191F677FB C4FB1C07E47C01B0D090BAC564FEAFBD0E24D90F01DE2B2E66A31E7012CACD42 30EA94EF415C8F2B1751F09BD8255A2C142CE2C8C69587EE6CE788273E55AFA7
Output Value:	15E3B39DA53DE7C20A60D3EC2DECC2C6B2DB65FE07B1188D666A8A8476E4910F 592FB3F8D49E4A01E5624FDF161A698EB0AA52515A79A46F3AFA1B8D7CEBB320
Status:	0: Normal

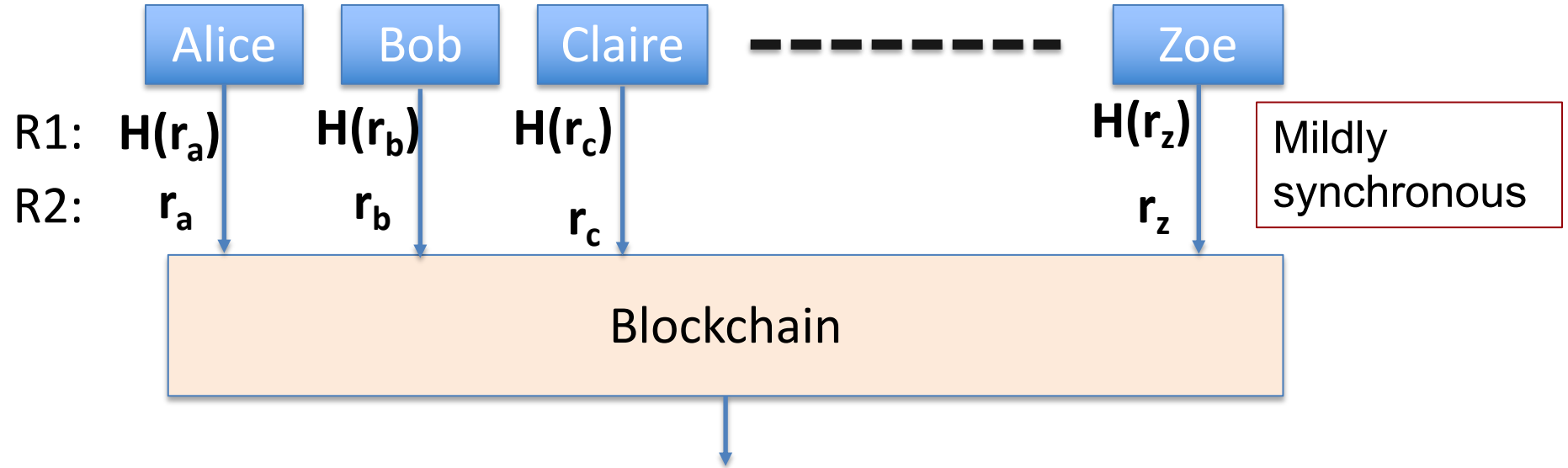
Collect randomness approach



output $\mathbf{beacon = Hash(r_a \,||\, r_b \,||\, \cdots \,||\, r_z)} \in \{0,1\}^{256}$

Problem: Zoe controls the final seed !!

Commit and Reveal



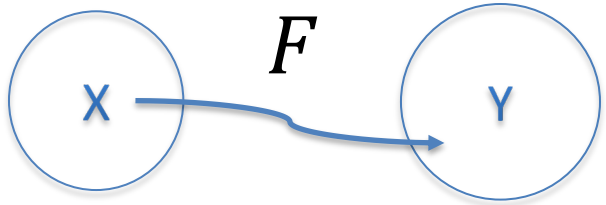
output **beacon = Hash(r_a || r_b || ⋯ || r_z)** $\in \{0,1\}^{256}$

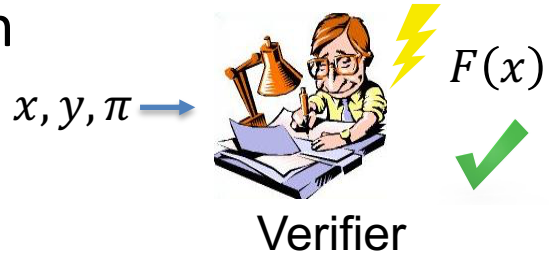
Problem: Beacon can be biased by not opening!!
K parties, k bits of influence

Verifiable Delay Function (VDF)

- **Function** – unique output for every input
- **Delay –** can be evaluated in time T

  cannot be evaluated in time (1-ϵ)T on parallel machine
- **Verifiable** – correctness of output can be verified efficiently





Security Properties (Informal)

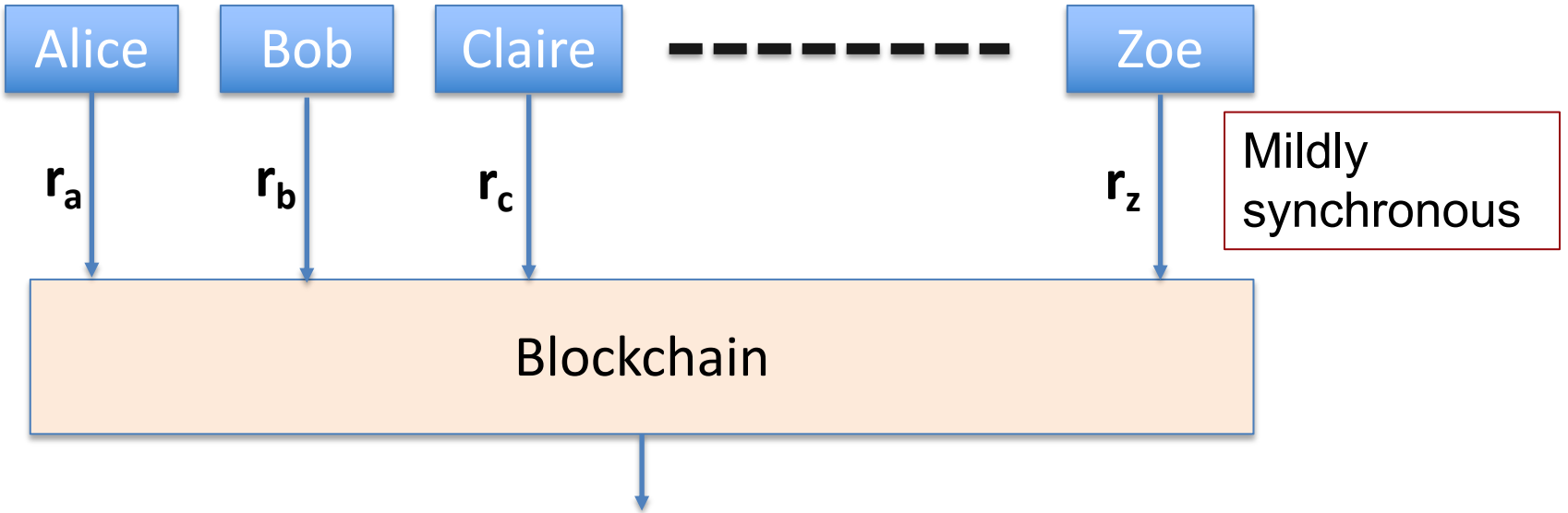
- Setup(λ, T) $\longrightarrow$ public parameters pp
- Eval(pp, $\boldsymbol{x}$) $\longrightarrow$ output $\boldsymbol{y}$, proof $\boldsymbol{\pi}$ (requires T steps)
- Verify(pp, $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{\pi}$) $\longrightarrow$ { *yes, no* }

"**Soundness**": if Verify(pp, x, $\boldsymbol{y}$, $\boldsymbol{\pi}$) = Verify(pp, x, $\boldsymbol{y'}$, $\boldsymbol{\pi'}$) = yes then $\boldsymbol{y = y'}$

"σ-**Sequentiality**": if A is a PRAM algorithm, time(A) $\leq \sigma(T)$,

e.g. $\sigma(T) = (1-\epsilon)T$ then Pr[$\boldsymbol{A(pp, x) = y}$] < negligible(λ)

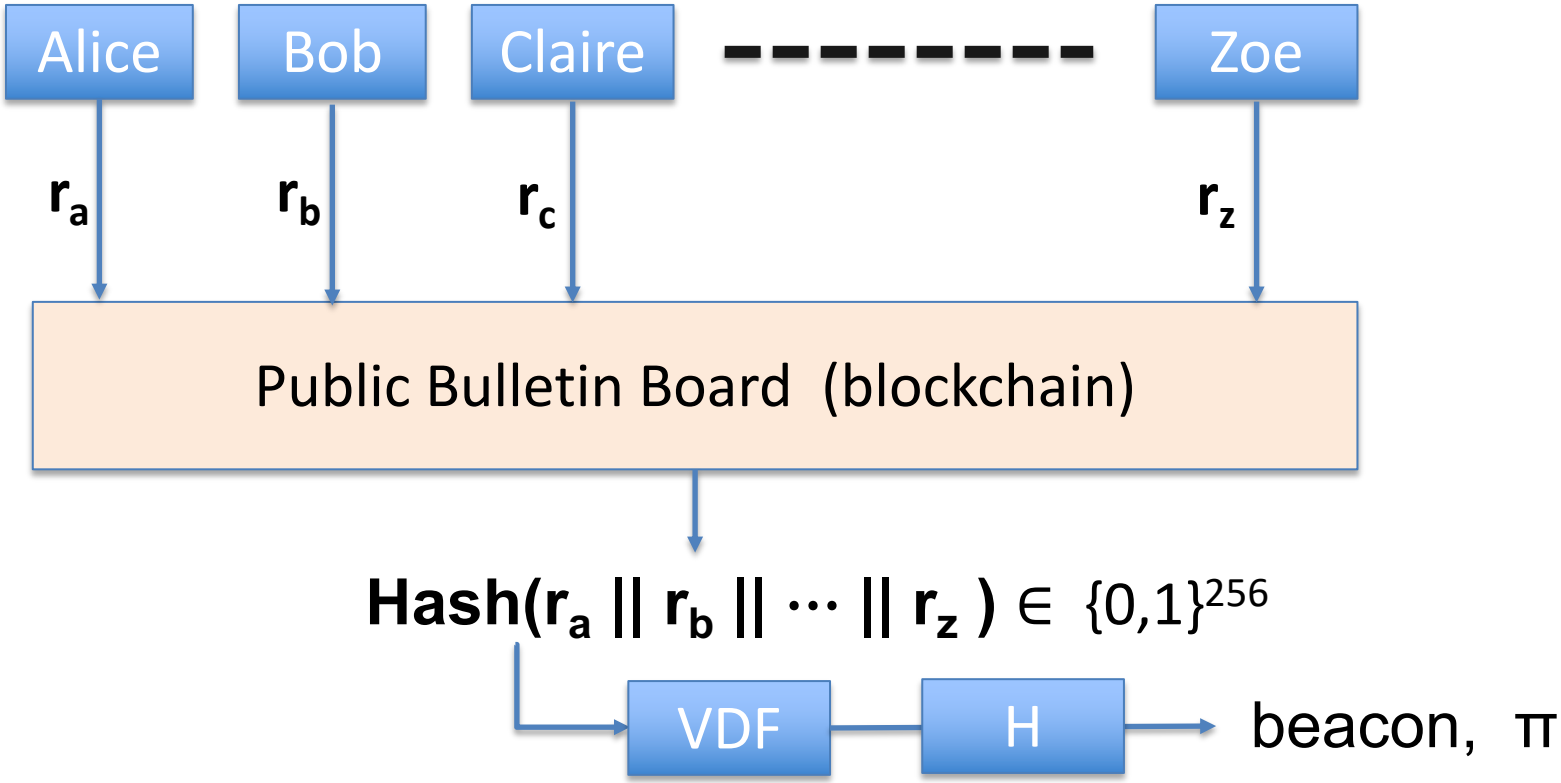
Collect randomness approach



output **beacon = Hash(r_a || r_b || ⋯ || r_z)** $\in \{0,1\}^{256}$

Problem: Zoe controls the final seed !!

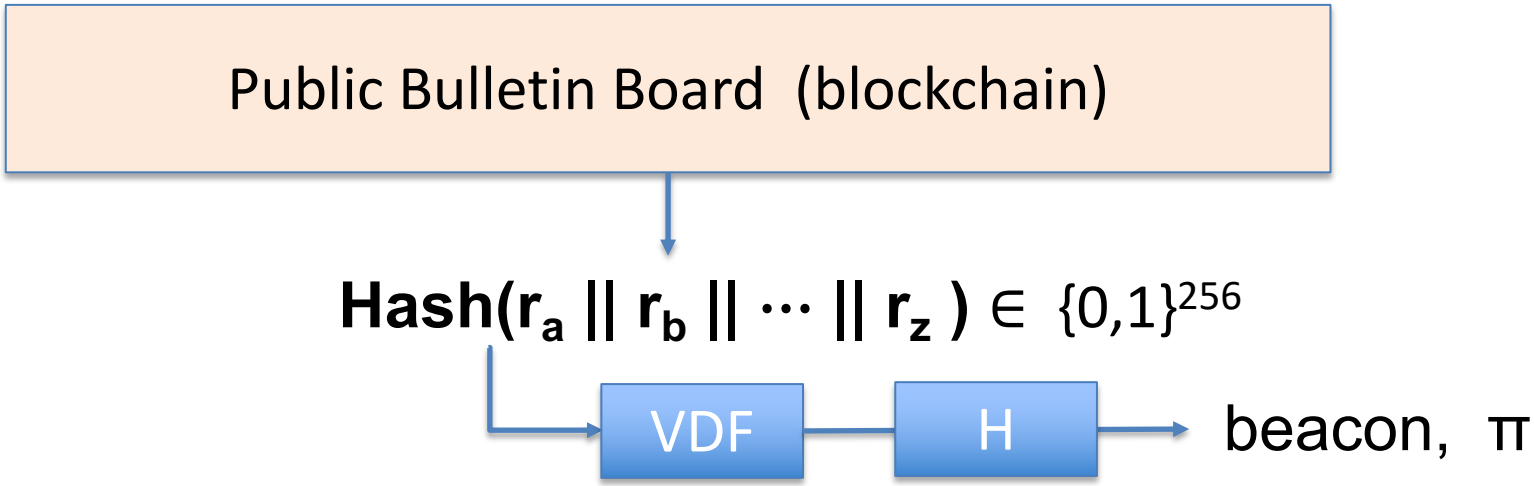
Solution: slow things down with a VDF [LW'15]



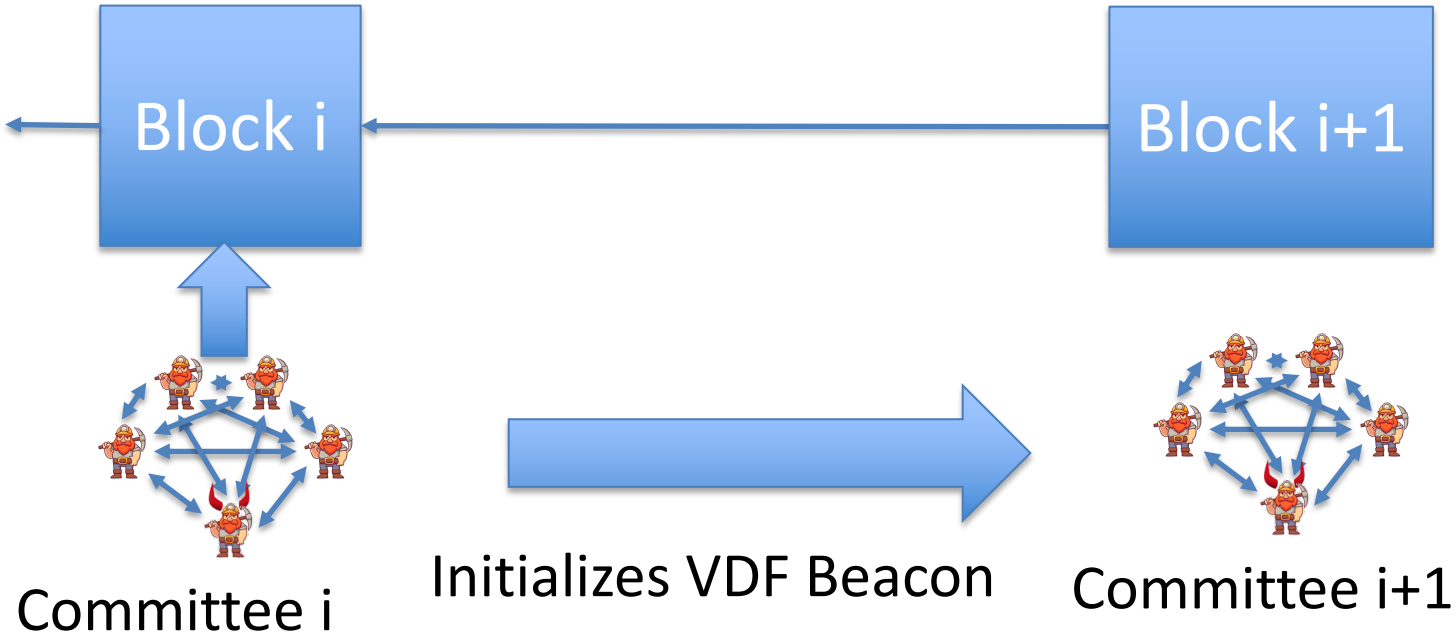
Solution: slow things down with a VDF [LW'15]

VDF delay $\gg$ max-Δ-time(Alice $\longrightarrow$ Zoe)

Uniqueness: ensures no ambiguity about output



VDF Beacon in a blockchain



How to build a VDF

Choose a “Group of unknown order”:

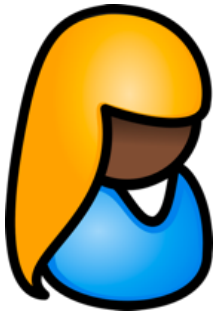
- Pick two primes p,q, Let $N = p \cdot q$
- Computing $g^{2^T} \; mod \; N$ takes T repeated squarings
  - Can’t be parallelized
  - Unless factorization of N is known
  - Taking roots mod N is hard!
- Let H be a hash function that maps to $[0, N-1]$

Eval(pp, x): output $H(x)^{2^T}$

How to verify?

VDF Proof [Wesolowski'18]

$(x, y, T): x^{2^T} = y$



y

Random λ bit prime l

$\log(T)$steps

Computes q,r s.t. $2^T = q \cdot l + r$ and $0 \le r < l$

$\pi = x^q$

Computes $r = 2^T \bmod l$

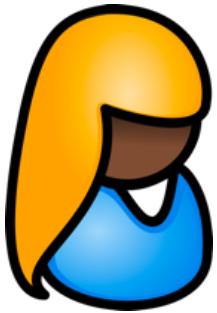
Checks:

$\pi^l x^r = y$

$x^{q \cdot l} x^r = x^{2^T}$

Security intuition

$(x, y, T): x^{2^T} = y$



y

Random λ bit prime l

Must compute π

s.t. $\pi = \left(\frac{y}{x^r}\right)^{\frac{1}{l}}$

Taking roots is hard

See reading

Computes q,r s.t. $2^T = q \cdot l + r$ and $0 \leq r < l$

$\pi = x^q$

Computes $r = 2^T \bmod l$

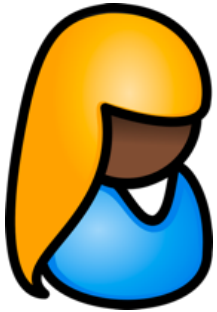
Checks:

$$\pi^l x^r = y$$

$$x^{q \cdot l} x^r = x^{2^T}$$

VDF Proof [Wesolowski'18]

$(x, y, T): x^{2^T} = y$



y

$l = H(x, y, T) \in Primes$

Computes
q,r s.t.
$2^T = q \cdot l + r$
and $0 \leq r < l$

$\pi = x^q, l$

Computes
$r = 2^T \ mod \ l$
Checks: $l = H(x, y, T)$
$\pi^l x^r = y$
$x^{q \cdot l} x^r = x^{2^T}$

END OF LECTURE

Next lecture:

Ethereum and Smart Contracts