SNARKs Lecture 5: Transparency, Recursive Proving

Instructor: Ben Fisch

Transparent SNARKs

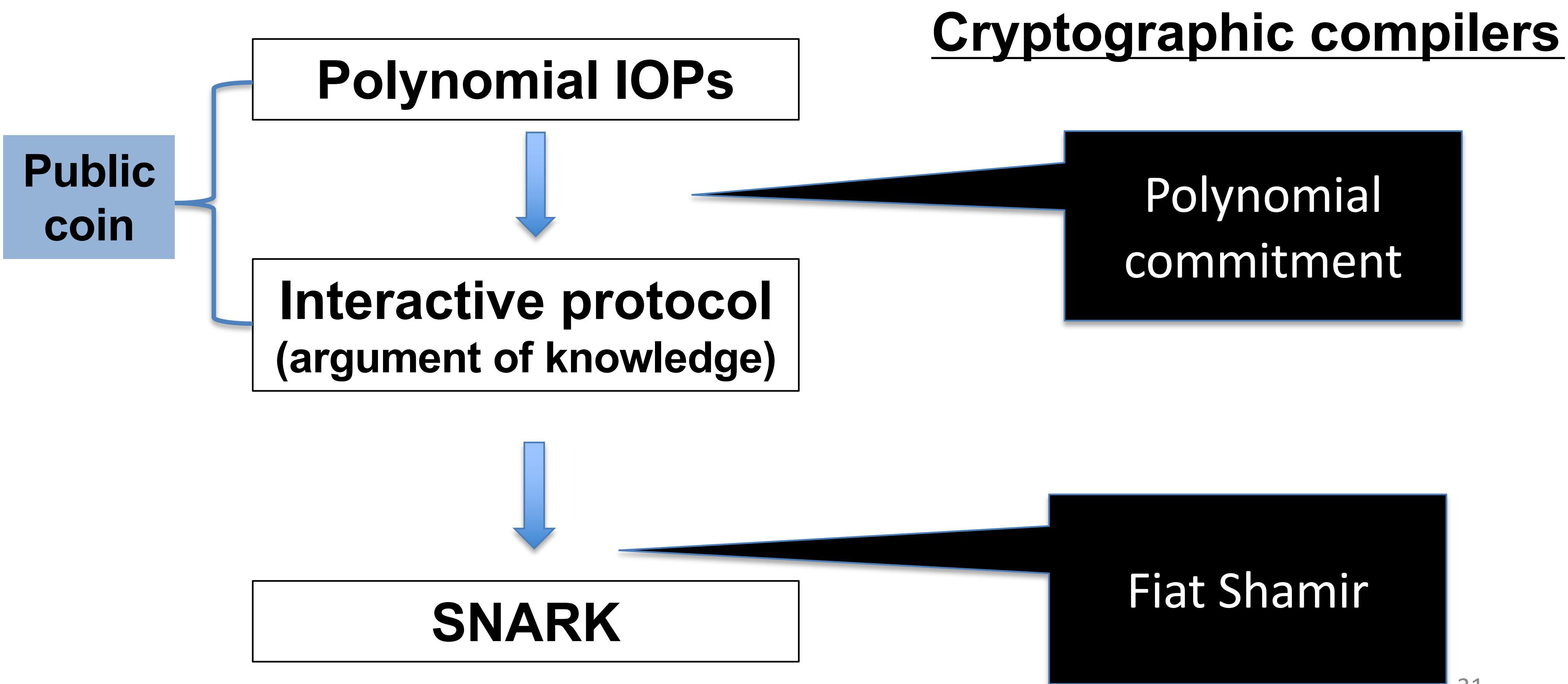
Problems with Trusted Setup

- If subverted, prover can create fake proof
- Can be alleviated through distributed setup
  - Expensive and difficult, but done for Zcash
- Low flexibility: New functionality → New setup
  - HAWK: Every smart contract has a new setup

IOPs with Succinct Queries

- Classical IOP (point queries)
- **Polynomial IOP**
  - Each oracle is a degree d polynomial f: $\mathbb{F}^{\mu} \rightarrow \mathbb{F}$
  - Verifier queries $q \in \mathbb{F}^{\mu}$ and receives $f(q)$

Polynomial IOP Compilation



IOP with Preprocessing

- A preprocessing phase establishes several oracles which are then used in future IOP instances
- The IOP verifier is given query access not only to oracles sent by the prover, but also oracles established in the preprocessing
- “Transparent setup”
  - Some of the IOP oracles may be derived from the R1CS program independent of the input or witness
  - These oracles may be preprocessed so that the verifiers (or public auditors) perform a one-time expensive operation to check the correctness of their full description.
  - Preprocessed oracles may then be queried cheaply in an IOP for the program on any input/witness pair (x, w).

Sonic: Polynomial IOP for NP

Theorem [MBKM19]:

- There exists a **2-round** polynomial* IOP for any NP relation R (with multiplicative complexity n) that makes **1 *bivariate*** and **6 *univariate*** *queries* to degree $2n$ polynomials . *Also preprocess degree 2n polynomials*.
- Transforms to **5-round** polynomial IOP with **39 *univariate*** oracle queries overall, degree $2n$

Marlin: Polynomial IOP for NP

Theorem [CHMMVW19]:

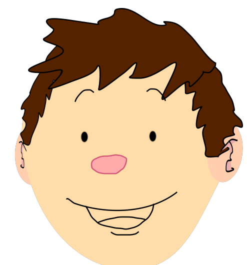
- There is a **4-round** polynomial IOP for any NP relation R (with R1CS complexity n) that makes **20 queries** ***at 3 distinct*** **query points** to 19 univariate polynomials (max degree 6n). The preprocessing checks 8 univariate degree n polynomials.

Polynomial Commitment

$f(X) \in \mathbb{F}_p[X]$ degree at most $\boldsymbol{d}$

C

small value



π

Small Proof that f(z) = y and deg(f)$\leq \boldsymbol{d}$

Polynomial Commitment

Input: $f(X) \in \mathbb{F}_p[X]$ degree at most $\boldsymbol{d}$

- **Setup**$(1^\lambda) \to pp$
- **Commit**$(pp, f) \to C$
- **Open** $(pp, C, f) \to b \in \{0,1\}$

(Interactive protocol)

- **Eval** $(pp, C, z, y, d; f)$ \\ Prover claim: exists *f(X)* s.t.
  \\ *f(z) = y* AND **Commit**(pp, *f*) = *C*

Efficiency: Succinctness

Input: $f(X) \in \mathbb{F}_p[X]$ degree at most $\boldsymbol{d}$

- **Setup**$(1^\lambda) \to pp$
- **Commit**$(pp, f) \to C$
- **Open** $(pp, C, f) \to b \in \{0,1\}$

|C| << f(X)

ideally O(λ)

Communication

sublinear in d

(Interactive protocol)

- **Eval** $(pp, C, z, y, d; f)$ \\\\ Prover claim: exists *f(X)* s.t.

\\\\ *f(z) = y* AND **Commit**(pp, *f*) = *C*

Security: Binding / Knowledge

Input: $f(X) \in \mathbb{F}_p[X]$ degree at most $\boldsymbol{d}$

- **Setup**$(1^\lambda) \rightarrow pp$
- **Commit**$(pp, f) \rightarrow C$
- **Open** $(pp, C, f) \rightarrow b \in \{0,1\}$

Standard commitment binding

(Interactive protocol)

Evaluation Binding / Argument of **Knowledge**

- **Eval** $(pp, C, z, y, d; f)$ \\ Prover claim: exists *f(X)* s.t.
  \\ *f(z) = y* AND **Commit**(pp, *f*) = *C*

Three Commitment Schemes

- FRI (BenSasson-Bentov-Horesh-Riabzev)
  - No trusted-setup, large proofs (>100KB), quantum secure
- DARK (Bünz-Fisch-Szepieniec)
  - No trusted–setup, smaller (8KB), not quantum secure
- Bilinear group commitment (Kate-Zaverucha-Golberg)
  - Trusted–setup, very small (32 Bytes), not quantum secure

**Size estimates for polynomial degree 1M

Three SNARK Schemes

- FRI-STARK (using FRI)
  - **No trusted-setup**, large proofs (>100KB), quantum secure
- Supersonic (using DARK)
  - **No trusted–setup**, smaller (~10KB), not quantum secure
- Sonic/PLONK/Marlin (using Bilinear group)
  - **One-time** trusted–setup, small (<1KB), not quantum secure

**Size estimates for circuit size 1M gates

Recursive SNARKs

Incremental Verifiable Computation

- Can we take a SNARK proof π for a statement C(x,w) = 0 and produce an "updated" π' that says $C(x, w) = 0 \wedge C(x', w') = 0$?
  - Naïve solution: redo entire SNARK for new combined statement
  - Provide both π' and π for new C(x', w')
  - Incremental SNARK: new statement that says C(x', w') is true and knowledge of π that proves C(x,w) = 0

“Recursive proofs”

“Proof of a proof”

- A proof π_1 that I know a proof π_0 that C(x,w) = 0

”proof of a proof of a proof…”

- A proof π_2 that I know a proof π_1 which proves knowledge of a proof π_0 that C(x,w) = 0.

SNARK of SNARK

$$S(C) \to S_P, S_V$$
$$\pi \leftarrow P(x, w)$$

Now write a circuit C' that verifies π:

- Input x' is x
- Witness w' is π
- $C'(x'$, w') = 0 iff V(S_V, π, x)=Accept

Finally:

$$S(C') \to S'_P, S'_V$$
$$\pi' \leftarrow P(x', w')$$

SNARK of SNARK…

- Note that C' depends only on V and S_V! Can represent code for verifier V as an R1CS program.
- We can also make C' more complex…
  - Input x' is x_0, x_1
  - Witness $w'$is π, w_1
  - $C'(x', w') = 0$ iff $V(S_V, \pi, x_0) = Accept$ AND $C(x_1, w_1) = 0$
- What about proving verification of π'? Needs new C'' that has S'_V hardcoded? Do we need to re-run setup for every additional level of recursion?

SNARK of SNARK…

- Do verifiers need to re-run setup for every additional level of recursion? **Proposed solution: make S_V part of the witness**
- Modify definition C_{i}':
    - Input x' is $x_0, x_1, \dots, x_i$
    - Witness w' is π, w_i, S_V
    - $C'(x', w') = 0$ iff $V(S_V, \pi, x_0, \dots, x_{i-1}) = Accept$ AND $\mathrm{S}(\mathrm{C}_{i-1}') \rightarrow (S_P, S_V)$ AND $C(x_i, w_i) = 0$
- Now C_i' may accept as witness a proof π that is a proof for C_{i-1}' generated using parameters $\left(S_P^{i-1}, S_V^{i-1}\right) \leftarrow S(C_{i-1}')$. *Prover still needs to re-run setup as part of proving.*

Universal SNARK Verifier

- What goes wrong when the setup is trusted? **Making S_V a witness does not work.**
  - S_V ”exists” even for proofs of false statements
  - Proof system only sound when prover does not know the secrets involved in the generation of S_V.
  - The existence of (S_V, π) that the algorithm V would accept is meaningless.
- New solution: **universal SNARK verifier**

Universal SNARK verifier

- UC is a circuit that takes inputs (C, x, w) and outputs $C(x, w)$
- $S(UC) \rightarrow (US_P, US_V)$ are parameters of SNARK system for UC
- Define C_i':
  - Input x' is $x_0, x_1, \ldots, x_i$
  - Witness w' is π, w_i, S_V
  - $C'(x', w') = 0$ iff $V(US_V, \pi, C_{i-1}', x_0, \ldots, x_{i-1}) = Accept$ AND AND $C(x_i, w_i) = 0$

Application: constant size blockchain

- **Recall roll-up application**: server processes 1000 txs and sends overall state change to blockchain along with SNARK proof of knowledge of the 1000 valid txs for the transition.
  - Blockchain content still grows linearly
  - Syncing with blockchain still requires verifying many historical SNARK proofs (every 1000 txs)
- **Constant-size blockchain:** Blockchain always has just one SNARK proof. Based on recursive proofs.
  - Upon each state transition, the blockchain proof is replaced with a <u>new SNARK</u> that proves both correctness of transition, and existence/correctness of previous SNARK.